Effect of nonlinear absorption on self-focusing of a laser beam in a plasma[a)]

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(Received 16 December 1977; accepted for publication 7 February 1978)

Considering both the real and imaginary parts of the dielectric constant to be intensity dependent, we have investigated the self-focusing of a Gaussian laser beam in a plasma. The mechanism or nonlinearity considered herein is the ponderomotive force or heating mainly determined by collisions of electrons with heavier particles. An equation for the beamwidth parameter and an expression of the axial intensity have been obtained in the WKB(J) and paraxial approximations. It is seen that the effect of nonlinear absorption on self-focusing is significant. Consideration of nonlinearity in absorption predicts focusing of a laser beam (under certain conditions) even when the linear-absorption approximation would predict defocusing of the beam. The results reduce to the corresponding analytical results reported earlier, when the intensity dependence of the dielectric constant is neglected. The technique adopted in the present investigation for solving the wave equation in the presence of nonlinear absorption is equally applicable to media other than a plasma.

PACS numbers: 42.65.Jx, 52.40.Db

I. INTRODUCTION

The phenomenon of self-focusing of laser beams has been intensively investigated in plasmas[1,2] as well as in other media.[3,4] In a plasma, the mechanisms responsible for this phenomenon are (1) relativistic variation[5] of the electron mass in a relativistic plasma, (2) ponderomotive force[1] and consequent redistribution of electrons and ions when the time scale of variation in the laser intensity is much smaller than the energy relaxation time of the plasma, (3) carrier redistribution[1] caused by nonuniform heating of electrons mainly determined by collisions when the time scale of variation in the laser intensity is much greater than the energy relaxation time of the plasma, and (4) carrier redistribution[1] caused by nonuniform heating of electrons limited mainly by thermal conduction in a collisional plasma.

Most of the available analyses[1,6] of self-focusing of laser beams in absorptive plasmas consider the intensity dependence of the real part ϵ_r of the dielectric constant, but assume the imaginary part ϵ_i of the dielectric constant to be a small ($\epsilon_i \ll \epsilon_r$) parameter independent of the laser intensity. However, the intensity dependence of ϵ_i is as strong as that of ϵ_r, and hence, the intensity dependence of ϵ_i should be taken into account. In a recent analytical investigation, Yuen[7] has taken this intensity dependence of ϵ_i into account. He has, however, assumed the dielectric constant to be independent of the distance of propagation and determined only by the initial intensity distribution of the laser beam. He has thus reduced the problem of self-focusing to the problem of focusing in a radially inhomogeneous optical waveguide.[8] Such an approximation is not valid in most cases. Apart from this drawback, his investigation is quite instructive and indirectly points out that the intensity dependence of ϵ_i can have a remarkable effect on the phenomenon of self-focusing.

[a)]Work supported by the National Science Foundation and the National Council of Educational Research and Training, India.

We have investigated here the effect of the intensity dependence of ϵ_i on self-focusing of a radially Gaussian laser beam in a plasma, by making use of the WKB(J) ($|d\ln K/d\xi| \ll qK_r$) and paraxial ($\rho \ll f$) approximations. In Sec. II, we have presented the expressions of the dielectric constant ϵ when its intensity dependence arises because of either the ponderomotive force or the collisional heating of electrons. In Sec. III, we have solved the wave equation for the electric field in the WKB(J) and paraxial approximations. A second-order differential equation for the beamwidth parameter f and an expression of the axial-intensity-amplification factor Ψ have been obtained by employing a slightly simplified version of the approach adopted by Akhmanov *et al.*[3] and Sodha *et al.*[1] In Sec. IV, we have presented numerical results along with a discussion.

We infer from the present investigation that the intensity dependence of the imaginary part ϵ_i of the dielectric constant is as strong as that of the real part ϵ_r. The effect of nonlinear absorption, represented by the intensity dependence of ϵ_i, on self-focusing of a laser beam in a plasma is quite significant. Consideration of nonlinearity in absorption predicts focusing of a laser beam even when the linear-absorption approximation predicts defocusing of the beam. The effect of nonlinear absorption is most significant in the case of a collisional plasma dominated by electron-diatomic molecule collisions ($S=2$). The results obtained by neglecting the intensity dependence of the dielectric constant of a radially inhomogeneous plasma are qualitatively constant with the corresponding analytical results obtained by Yuen.[7] The technique adopted in the present investigation for solving the wave equation in the presence of nonlinear absorption is equally applicable to media[4] other than a plasma.

II. DIELECTRIC CONSTANT

The dielectric constant of an unmagnetized plasma at frequency ω much greater than the "effective" electron-collision frequency ν is given by[1,9]

$$\epsilon = 1 - WP - iVQ. \qquad (2.1)$$

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where

$$W = 4\pi N_0 e^2/m\omega^2, \quad (2.2)$$

$$V = W\nu_0/\omega. \quad (2.3)$$

m is the electron rest mass, N_0 is the electron concentration in the absence of the field, and ν_0 is the electron-collision frequency in the absence of the field.

$$P = N/N_0 \quad (2.4)$$

represents the field-induced variation in the electron concentration N,

$$Q = N\nu/N_0\nu_0 = P^b(T/T_0)^{S/2} \quad (2.5)$$

represents the field-induced variation[1,9] in the product $N\nu$ of the electron concentration and the electron-collision frequency, T is the electron temperature, $T_0 = T$ in the absence of the field, and

$$b = 2 \text{ or } 1, \quad (2.6)$$

depending upon whether the electron collisions are with ions or neutrals,[1,9]

$$S = -3 \text{ or } 1 \text{ or } 2, \quad (2.7)$$

depending upon whether the electron collisions are with ions or nondiatomic molecules or diatomic molecules. We have assumed the plasma to be nonrelativistic so that the electron mass[5] remains unaffected by the laser beam.

It is convenient to use the dimensionless intensity

$$I = \beta \mathbf{E}^* \cdot \mathbf{E}, \quad (2.8)$$

where the normalization constant β depends upon the mechanism of nonlinearity under consideration. It can be easily shown that

$$\beta = \text{arbitrary}, \quad (2.9a)$$

$$P = 1, \quad (2.10a)$$

$$Q = 1, \quad (2.11a)$$

in the absence of nonlinearity[7,8];

$$\beta = e^2/4mK_BT_0\omega^2, \quad (2.9b)$$

$$P = \exp(-I), \quad (2.10b)$$

$$Q = \exp(-bI), \quad (2.11b)$$

in the case of a ponderomotive force mechanism[1,2]; and

$$\beta = Me^2/6m^2K_BT_0\omega^2, \quad (2.9c)$$

$$P = (1+I)^{S/2-1}, \quad (2.10c)$$

$$Q = (1+I)^{bS/2-b}(1+2I)^{S/2}, \quad (2.11c)$$

in the case of a collisional heating mechanism.[1] In Eq. (2.9c), M is the rest mass of ions/neutrals, K_B is Boltzmann's constant, and e is the electronic charge.

For a slightly diverging/converging beam, the nature of the radial-intensity profile of the laser beam remains[1,3] unaltered (in the paraxial region) in the course of its propagation. Hence, for an initially radially Gaussian beam, I is given by

$$I = I_a \exp(-r^2/r_0^2 f^2) = I_a \exp(-h_r \rho^2), \quad (2.12)$$

where the axial intensity I_a and the beamwidth parameter

$$f = h_r^{-1/2} \quad (2.13)$$

vary with the distance of propagation z, but not with the radial coordinate

$$r \equiv \rho r_0. \quad (2.14)$$

Expression (2.12) is consistent with expression (3.7) of the electric field E. Using Eqs. (2.1) and (2.12) in the paraxial region, we obtain[2]

$$\epsilon \simeq (\epsilon_{ar} - i\epsilon_{ai}) - (\epsilon_{2r} + i\epsilon_{2i}) h_r \rho^2, \quad (2.15)$$

where

$$\epsilon_{ar} = 1 - W_a P_a, \quad (2.16)$$

$$\epsilon_{ai} = V_a Q_a, \quad (2.17)$$

$$\epsilon_{2r} = W_b P_a/h_r + W_a P_2, \quad (2.18)$$

$$\epsilon_{2i} = V_b Q_a/h_r + V_a Q_2, \quad (2.19)$$

and we have used the notations

$$W_a = W(\rho = 0), \quad (2.20)$$

$$V_a = V(\rho = 0), \quad (2.21)$$

$$W_b = \left(\frac{dW}{d\rho^2}\right)(\rho = 0), \quad (2.22)$$

$$V_b = \left(\frac{dV}{d\rho^2}\right)(\rho = 0), \quad (2.23)$$

$$P_a = P(\rho = 0) = P(I = I_a), \quad (2.24)$$

$$Q_a = Q(\rho = 0) = Q(I = I_a), \quad (2.25)$$

$$P_2 = \left(\frac{dP}{d\rho^2}\right)(\rho = 0) = -I_a\left(\frac{dP_a}{dI_a}\right), \quad (2.26)$$

$$Q_2 = \left(\frac{dQ}{d\rho^2}\right)(\rho = 0) = -I_a\left(\frac{dQ_a}{dI_a}\right). \quad (2.27)$$

For convenience, let us write down the expressions of P_a, Q_a, P_2, and Q_2 explicitly (cf. Fig. 1).

$$P_a = 1, \quad (2.28a)$$

$$Q_a = 1, \quad (2.29a)$$

$$P_2 = 0, \quad (2.30a)$$

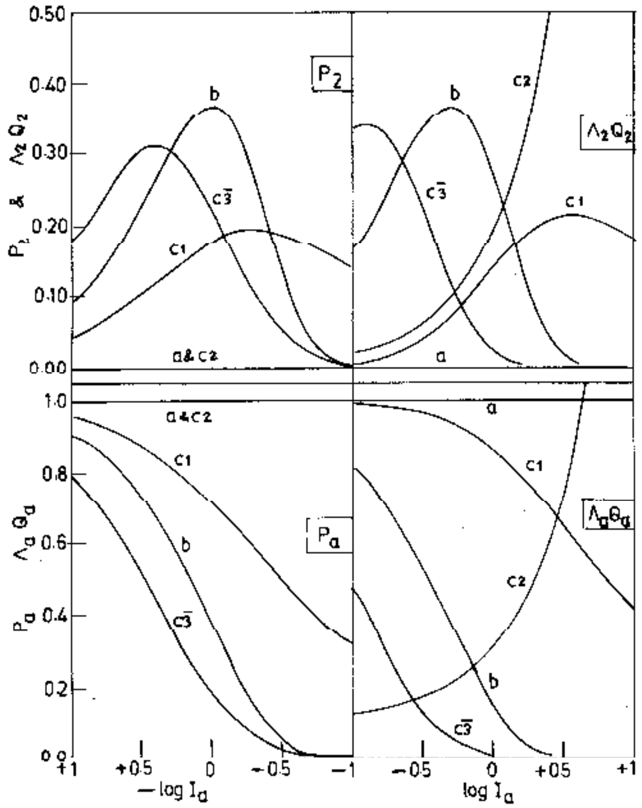

FIG. 1. The functions P_a, $\Lambda_a Q_a$, P_2, and $\Lambda_2 Q_2$ versus $\log I_a$ in (a) the absence of nonlinearity, (b) the case of a ponderomotive force mechanism with $b=2$, and (c) the case of a collisional heating mechanism with $S=-3$ for (c$\bar{3}$), 1 for (c1), and 2 for (c2). $\Lambda_a=1$ for (a), (b), (c$\bar{3}$), and (c1) and 0.1 for (c2). $\Lambda_2=1$ for (a), (b), (c$\bar{3}$), and (c1) and -0.1 for (c2).

$$Q_2=0, \tag{2.31a}$$

in the absence of nonlinearity;

$$P_a=\exp(-I_a), \tag{2.28b}$$

$$Q_a=\exp(-bI_a), \tag{2.29b}$$

$$P_2=I_a P_a, \tag{2.30b}$$

$$Q_2=bI_a Q_a, \tag{2.31b}$$

in the case of a ponderomotive force mechanism; and

$$P_a=(1+I_a)^{S/2-1}, \tag{2.28c}$$

$$Q_a=(1+I_a)^{bS/2-b}(1+2I_a)^{S/2}, \tag{2.29c}$$

$$P_2=(1-\tfrac{1}{2}S)(1+I_a)^{-1}I_a P_a, \tag{2.30c}$$

$$Q_2=[(\tfrac{1}{2}bS-b)(1+I_a)^{-1}-S(1+2I_a)^{-1}]I_a Q_a, \tag{2.31c}$$

in the case of a collisional heating mechanism.

III. BEAM PROPAGATION

When $k_0^2 \gg |\nabla^2 \ln\epsilon|$, a plane polarized electric field in a cylindrically symmetric plasma is governed by the scalar wave equation[1,3]

$$\frac{\partial^2 E}{\partial \xi^2}+\frac{q}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial E}{\partial\rho}+q^2\epsilon E=0, \tag{3.1}$$

where we have used the dimensionless coordinates

$$\xi=z/k_0 r_0^2, \tag{3.2}$$

$$\rho=r/r_0, \tag{3.3}$$

and the notations

$$q=k_0^2 r_0^2, \tag{3.4}$$

$$k_0=\omega/c. \tag{3.5}$$

As a boundary condition, we take the initial electric field to be given by

$$E(\xi=0)=E_0\exp(i\omega t-\tfrac{1}{2}\rho^2). \tag{3.6}$$

If the beam diverges/converges very slowly, we may express the solution of Eq. (3.1) in the paraxial region in the form[1,3]

$$E=E_0\exp\left(i\omega t-iq\int_0^\xi K\,d\xi+\tfrac{1}{2}g-\tfrac{1}{2}h\rho^2\right), \tag{3.7}$$

where

$$K=(\epsilon_{0r}-i\epsilon_{0i})^{1/2}$$
$$=K_r-iK_i, \tag{3.8}$$

$$g=g_r+ig_i, \tag{3.9}$$

$$h=h_r+ih_i. \tag{3.10}$$

Substituting for E from Eq. (3.7) in Eq. (3.1), and then equating the real and imaginary coefficients[1,3] of ρ^0 and ρ^2 on both sides of the resulting equation, we obtain four coupled equations. In the WKB(J) approximation, we neglect the second-order terms in Eqs. (3.7)–(3.10). Moreover, assuming

$$K_r \gg K_i, \tag{3.11}$$

we may also neglect the terms containing K_i. Equations (3.7)–(3.10) then reduce to

$$K_r\frac{dg_i}{d\xi}-2h_r=0, \tag{3.12}$$

$$K_r\frac{dg_r}{d\xi}+2h_i+\frac{dK_r}{d\xi}=0, \tag{3.13}$$

$$K_r\frac{dh_i}{d\xi}-h_r^2+h_i^2+q\epsilon_{2r}h_r=0, \tag{3.14}$$

$$K_r\frac{dh_r}{d\xi}+2h_r h_i-q\epsilon_{2i}h_r=0. \tag{3.15}$$

Equation (3.15) leads to

$$h_i=\frac{q\epsilon_{2i}}{2}-\frac{K_r}{2h_r}\frac{dh_r}{d\xi}. \tag{3.16}$$

Substituting for h_i in Eq. (3.14), we then obtain

$$\frac{d^2h_r}{d\xi^2}-\frac{3}{2h_r}\left(\frac{dh_r}{d\xi}\right)^2+\left(\frac{1}{K_r}\frac{dK_r}{d\xi}+\frac{q\epsilon_{2i}}{K_r}\right)\frac{dh_r}{d\xi}+\left(\frac{2h_r^3}{K_r^2}-\frac{2q\epsilon_{2r}h_r^2}{K_r^2}-\frac{q^2\epsilon_{2i}^2 h_r}{2K_r^2}-\frac{q}{K_r}\frac{d\epsilon_{2i}}{d\xi}h_r\right)=0. \tag{3.17}$$

Using Eq. (2.13), we then obtain the following equation for the beamwidth parameter f:

$$\frac{d^2f}{d\xi^2}+\left(\frac{1}{K_r}\frac{dK_r}{d\xi}+\frac{q\epsilon_{2i}}{K_r}\right)\frac{df}{d\xi}$$
$$=\left(\frac{1}{K_r^2f^3}-\frac{q\epsilon_{2r}}{K_r^2f}-\frac{q^2\epsilon_{2i}^2f}{4K_r^2}-\frac{q}{2K_r}\frac{d\epsilon_{2i}}{d\xi}f\right). \qquad (3.18)$$

For an initially plane wave front, we have

$$f(\xi=0)=1, \qquad (3.19)$$

$$\left(\frac{df}{d\xi}\right)_{(\xi=0)}=0. \qquad (3.20)$$

To solve Eq. (3.18) in the presence of nonlinearity (i.e., intensity dependence of the dielectric constant), we need an expression for the axial intensity I_a. It can be easily seen that

$$I_a=\Psi I_0, \qquad (3.21)$$

where the axial-intensity-amplification factor Ψ is given by

$$\Psi=\exp(-2q\int_0^\xi K_i\,d\xi+g_r), \qquad (3.22)$$

and the incident axial intensity I_0 is

$$I_0=\beta E_0^2. \qquad (3.23)$$

Using Eqs. (3.13), (3.16), and (3.22) we obtain

$$\Psi=(K_{r0}/K_rf^2)\exp(-2q\int_0^\xi K_{i\,\mathrm{eff}}\,d\xi), \qquad (3.24)$$

where

$$K_{r0}=K_r(\xi=0) \qquad (3.25)$$

and

$$K_{i\,\mathrm{eff}}=K_i+\epsilon_{2i}/2K_r. \qquad (3.26)$$

In the case of a nonabsorption plasma which is homogeneous in the absence of the field, Eqs. (3.18) and (3.26) reduce to[2]

$$\frac{d^2f}{d\xi^2}+\frac{1}{K_r}\frac{dK_r}{d\xi}\frac{df}{d\xi}=\left(\frac{1}{K_r^2f^3}-\frac{qW_aP_2}{K_r^2f}\right) \qquad (3.27)$$

and

$$K_{i\,\mathrm{eff}}=0. \qquad (3.28)$$

For $I_0 \leq I_L$ ($\simeq 1/qW_a$ in the case of a ponderomotive force mechanism), f increases (i.e., Ψ decreases) monotonically with ξ. For $qW_a>(qW_a)_{cr}$ [$=\exp(1)$ in the case of a ponderomotive force mechanism], the equation

$$P_2(I_a=I_{st})=1/qW_a \qquad (3.29)$$

has two roots for I_{st}, say $I_{st\,l}$ and $I_{st\,u}$. For $I_0=I_{st\,l}$ or $I_{st\,u}$, $f=\Psi=1$ for all ξ. For $I_{st\,l}<I_0<I_{st\,u}$, f (i.e., Ψ) oscillates periodically with ξ, between its initial value of unity and a minimum value $f_{min}<1$ (i.e., maximum value $\Psi_{max}>1$). For $I_L<I_0<I_{st\,l}$ or $I_0>I_{st\,u}$, f(i.e., Ψ) oscillates periodically with ξ, between the initial value of unity and a maximum value $f_{max}>1$ (i.e., minimum value $\Psi_{min}<1$).

In the case of a nonabsorptive plasma which is radially homogeneous but axially inhomogeneous in the absence of the field, Eqs. (3.27) and (3.28) remain valid. However, explicit ξ dependence of K_r and W_a leads to growth or decay of variations in f and Ψ depending upon whether W_a decreases or increases with ξ; the foregoing WKB(J) analysis becomes invalid in the vicinity of the resonant layer and beyond, where $W_aP_a \lesssim 1$.

In the absence of nonlinearity in a medium which is radially inhomogeneous,[8] Eqs. (3.18) and (3.26) reduce to

$$\frac{d^2f}{d\xi^2}+\left(\frac{1}{K_r}\frac{dK_r}{d\xi}+\frac{2qV_bf^2}{K_r}\right)\frac{df}{d\xi}$$
$$=\left(\frac{1}{K_r^2f^3}-\frac{qW_bf}{K_r^2}-\frac{q^2V_b^2f^5}{4K_r^2}-\frac{q}{2K_r}\frac{dV_b}{d\xi}f^3\right) \qquad (3.30)$$

and

$$K_{i\,\mathrm{eff}}=K_i+V_bf^2/2K_r. \qquad (3.31)$$

In the absence of axial inhomogeneity and when $dV_b/d\xi=0$, the beam focuses or defocuses depending upon whether $(qW_b+\frac{1}{4}q^2V_b^2)$ is $>$ or <1, and the variations in f grow or decay with ξ depending upon whether $V_b<$ or >0. This case has been extensively investigated, mostly analytically, as a part of fiber optics[8] and also as an approximation to the self-focusing in a plasma.[7] Analytical results[7] predict that the oscillations in f are periodic if $dW/d\xi=V_b=0$, even when $K_i\neq 0$. Equation (3.30) is consistent with this analytical result. Occurrence of any term containing K_i in Eq. (3.18) would have made the equation inconsistent with this analytical result. (cf. Figs 2 and 3).

Under the linear-absorption approximation[1,6] in the case of an absorptive plasma which is homogeneous in the absence of the field, Eq. (3.27) remains valid, but Eq. (3.28) is replaced by

$$K_{i\,\mathrm{eff}}=K_i(\xi=0,\ Q_a=1)$$
$$\equiv K_{i0}. \qquad (3.32)$$

The effect of linear absorption is (1) to decrease the value of $(\Psi_{max}+\Psi_{next\text{-}min})$ monotonically as ξ increases, (2) to enhance the rate of monotonic rise in f (i.e., fall in Ψ) when $I_0<I_L$, (3) to render self-focusing of a laser beam impossible even when $I_0=I_{st}$ initially, and (4) to damp out the variations in f and Ψ until the value of Ψ becomes so small that f starts increasing (i.e., Ψ starts decreasing) monotonically with ξ. In the last mentioned effect, the number of oscillations in f that can occur (before f starts increasing monotonically) increases as the incident axial intensity I_0 or the scale length $(qK_{i0})^{-1}$ of absorption is increased.

In the case of a nonlinearly absorbing plasma, which is homogeneous in the absence of the field, Eqs. (3.27) and (3.28) are replaced by

$$\frac{d^2f}{d\xi^2}+\left(\frac{1}{K_r}\frac{dK_r}{d\xi}+\frac{qV_aQ_2}{K_r}\right)\frac{df}{d\xi}$$
$$=\left(\frac{1}{K_r^2f^3}-\frac{qW_aP_2}{K_r^2f}-\frac{q^2V_a^2Q_2^2f}{4K_r^2}-\frac{qV_a}{2K_r}\frac{dQ_2}{d\xi}f\right) \qquad (3.33)$$

and

$$K_{i\,\mathrm{eff}}=K_i+V_aQ_2/2K_r. \qquad (3.34)$$

Thus, we find that the effect of nonlinear absorption is (1) to increase or decrease the net absorption rate, (2) to increase or decrease the rate of ξ-dependent variation in f and Ψ depending upon whether $K_{i\,\mathrm{eff}}$ is $>$or $<K_{i0}$, and (3) to make focusing of the laser beam possible even when the linear-absorption approximation would predict defocusing of the beam. (cf. Figs. 4–8).

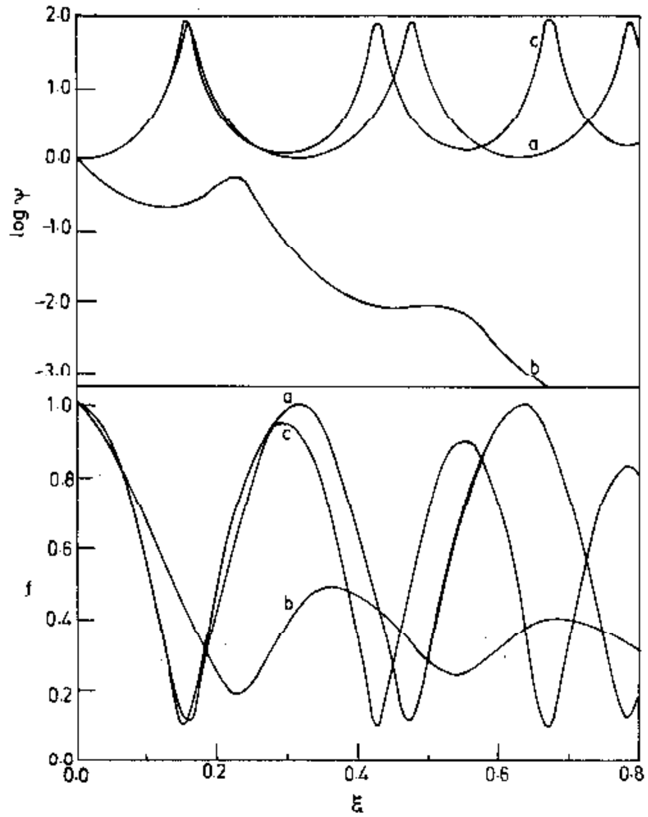

FIG. 2. f and $\log\psi$ versus ξ in the absence of nonlinearity for $q=900$, $W=0.1(1+\rho^2)$ for (a) and (b) and $0.1(1+\xi)(1+\rho^2)$ for (c); $V=0$ for (a) and (b) and $0.01\,(1+2\rho^2)$ for (b).

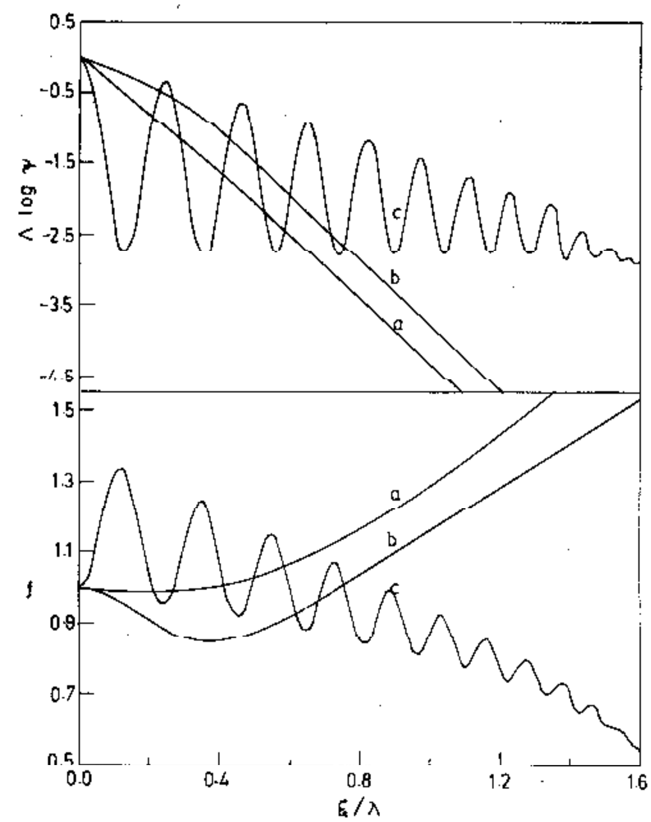

FIG. 4. f and $\Lambda\log\psi$ versus ξ/λ in the case of a ponderomotive force mechanism with $b=2$ for $I_0=0.1$ for (a), 1 for (b), and 10 for (c); $q=900$, $W=0.1$, $V=0.01$ for (a), 0.1 for (b), and 1 for (c), $\lambda=1$ for (a), 0.1 for (b), and 10 for (c); $\Lambda=1$ for (a) and (b) and 10 for (c).

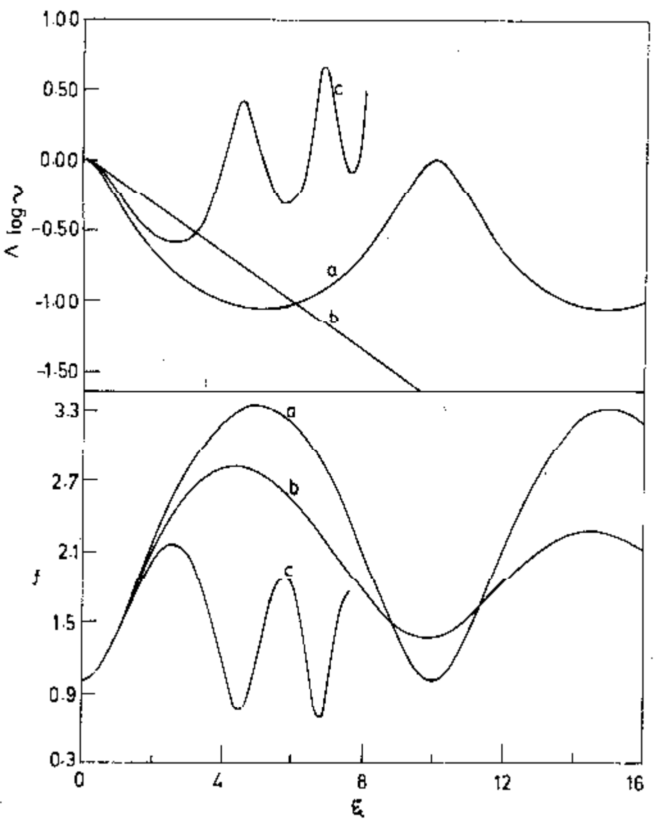

FIG. 3. f and $\Lambda\log\psi$ versus ξ in the absence of nonlinearity for $q=900$, $W=0.1(1+\rho^2/1000)$ for (a) and (b) and $0.1(1+\xi)(1+\rho^2/1000)$ for (c), $V=0$ for (a) and (c) and $0.01(1+2\rho^2/1000)$ for (b). $\Lambda=1$ for (a) and (c) and 0.04 for (b).

In the case of a nonlinearly absorbing plasma, which is radially homogeneous but axially inhomogeneous in the absence of the field, the right-hand side of Eq. (3.33) is reduced by a term $(q/2K_r)(dV_a/d\xi)Q_2 f$, whereas Eq. (3.34) remains valid. Explicit ξ dependence of K_r, W_a, and V_a increases or decreases the rate of the aforementioned absorption-induced damping in the variations in f depending upon whether W_a increases or decreases with ξ.

IV. RESULTS AND DISCUSSION

Figure 1 shows the dependence of the functions P_a, Q_a, P_2, and Q_2 on the axial intensity I_a in the (a) absence of nonlinearity,[7] (b) case of a ponderomotive force mechanism,[1] and (c) case of a collisional heating mechanism.[1] Figure 1 is very helpful in determining the extent of focusing and absorption of the beam and growth/decay of variation in f. Figure 1 shows that the intensity dependence of the imaginary part ϵ_i of the dielectric constant is as strong as that of the real part ϵ_r, and hence the linear-absorption approximation is doubtful. In the case of a collisional heating mechanism with $S=2$, ϵ_r is intensity independent, whereas Q_a and Q_2 (which occur in the expression of ϵ_i) increases and decreases linearly with I_a.

We have solved Eq. (3.18) for the beamwidth parameter f by using the Runge-Kutta method and evaluated the

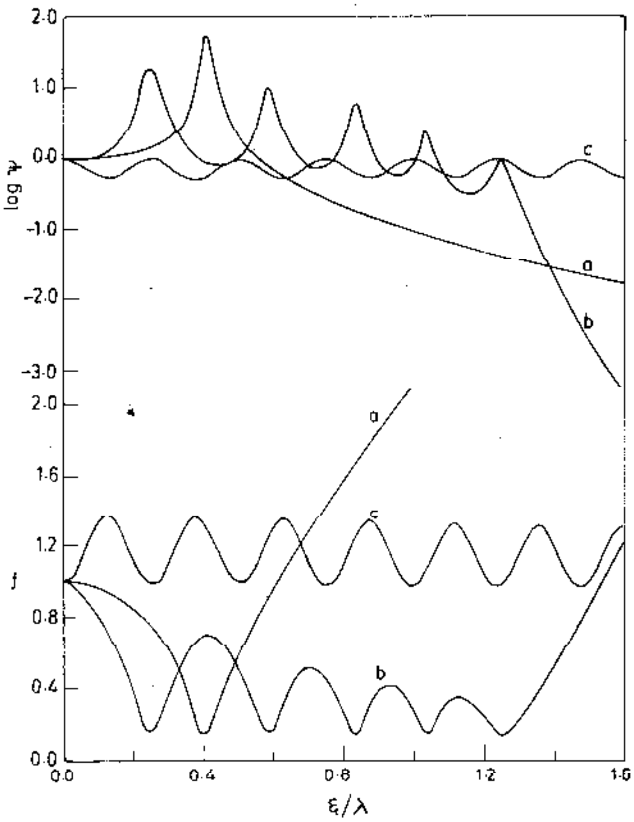

FIG. 5. f and $\log\psi$ versus ξ/λ in the case of a ponderomotive force mechanism with $b=2$ for $I_0=0.1$ for (a), 1 for (b), and 10 for (c); $q=900$, $W=0.1$, $V=0.001$ for (a), 0.01 for (b), and 0.1 for (c). $\lambda=1$ for (a) and (b) and 10 for (c).

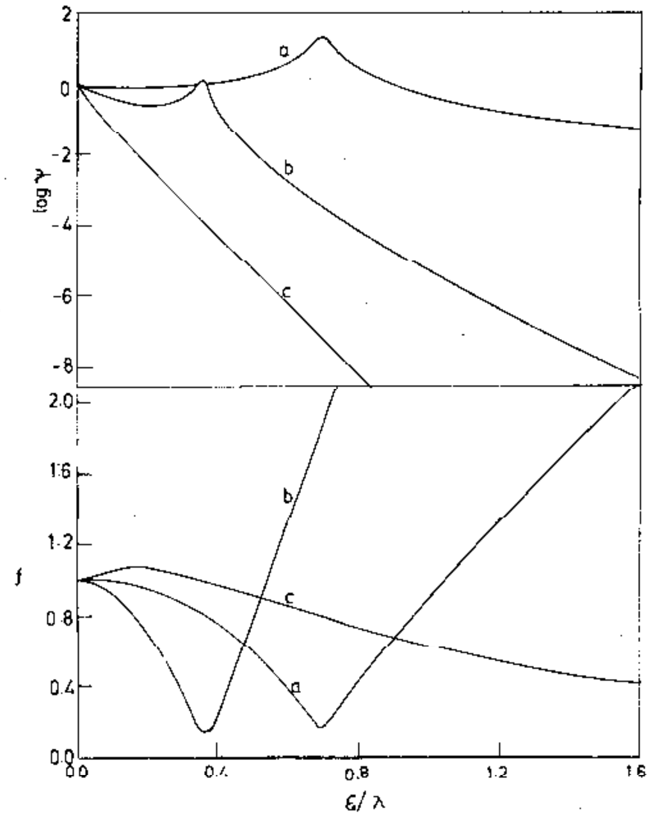

FIG. 7. f and $\log\psi$ versus ξ/λ in the case of a collisional heating mechanism with $S=1$ for $I_0=0.1$ for (a), 1 for (b), and 10 for (c); $q=900$, $W=0.1$, $V=0.001$ for (a), 0.01 for (b), and 0.1 for (c). $\lambda=1$ for (a) and (b) and 0.25 for (c).

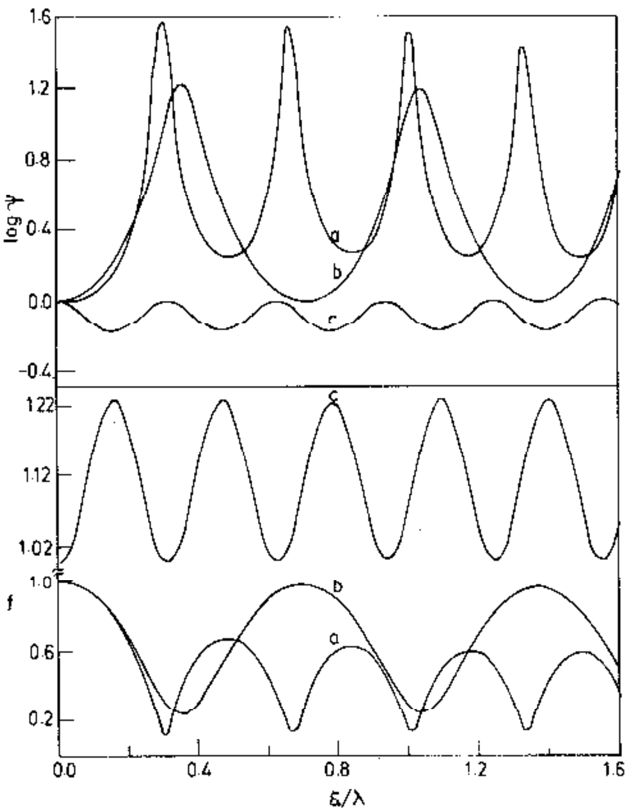

FIG. 6. f and $\log\psi$ versus ξ/λ in the case of a collisional heating mechanism with $S=-3$ for $I_0=0.1$ for (a), 1 for (b), and 10 for (c); $q=900$, $W=0.1$, $V=0.001$ for (a), 0.01 for (b), and 0.1 for (c). $\lambda=1$ for (a) and (b) and 10 for (c).

axial-intensity-amplification factor Ψ given by Eq. (3.24).

Figures 2 and 3 represent the variations of the beam-width parameter f and the axial-intensity-amplification factor Ψ with ξ in the absence of nonlinearity.[7] Figure 2 is for such a large value of W_0 that $f<1$, whereas Fig. 3 is for such a small value of W_0 that $f>1$. In both Figs. 2 and 3, we have considered three typical cases: (a) absence of both absorption and axial inhomogeneity, (b) radially increasing absorption without axial inhomogeneity, and (c) axially increasing concentration without absorption. The results presented here are consistent with the analytical result.[7,8] It is, moreover, seen that decay in the variations in f can occur because of axially increasing concentration, as well as because of radially increasing absorption.

Figures 4 and 5 represent the parametric variations of f and Ψ with ξ corresponding to the ponderomotive force mechanism[1] in a plasma which is homogeneous in the absence of the field. The value of V (which determines the magnitude of absorption of a laser beam of a given intensity) has been taken to be larger in Fig. 4 than in Fig. 5. These figures confirm the effect of absorption discussed in Sec. III. In addition, it is seen that (1) stabilization in f does not necessarily imply stabilization in Ψ and (2) $(-df_{max}/d\xi)>(-df_{min}/d\xi)$ and $(-d\Psi_{max}/d\xi)>(-d\Psi_{min}/d\xi)$ when the variations in f (and Ψ) damp with ξ.

Figures 6–8 represent the parametric variations of

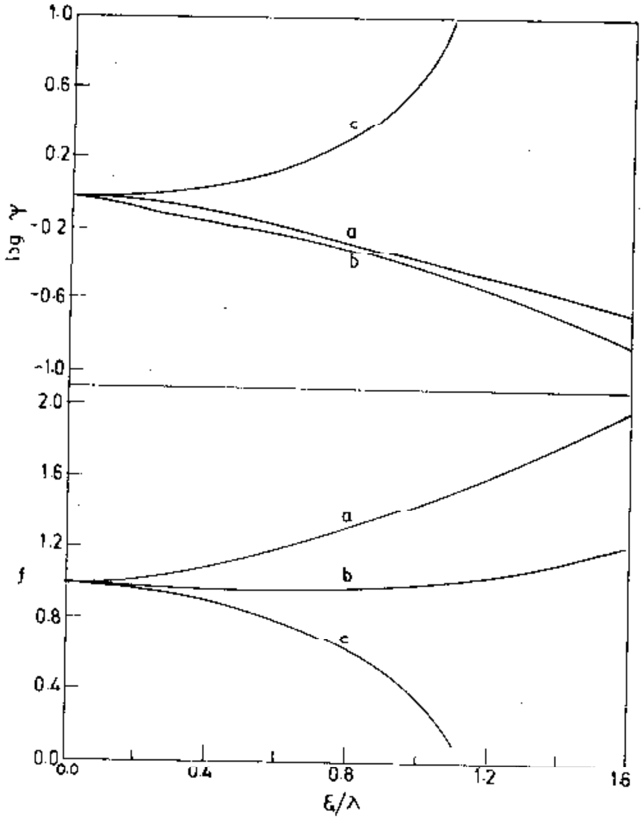

FIG. 8. f and $\log\psi$ versus ξ/λ in the case of a collisional heating mechanism with $S=2$ for $I_0=0.1$ for (a), 1 for (b), and 10 for (c); $\alpha=900$, $W=0.1$, $V=0.0001$ for (a), 0.001 for (b), and 0.01 for (c). $\lambda=1$ for (a) and (b) and 0.01 for (c).

f and Ψ with ξ corresponding to the collisional heating mechanism[1] with $S=-3$, 1, and 2, respectively, in a plasma which is homogeneous in the absence of the field. Results in the cases of Figs. 6 and 7 are similar to the results in the case of Fig. 5; this is expected from the similarity in the expressions of P_a, Q_a, P_2, and Q_2. The effect of the increase in the value of S is seen to be quite appreciable; it (1) reduces the number of oscillations in f that can occur before f starts increasing, (2) distorts the pattern of oscillations in f, and (3) increases the values of $K_{i\,\mathrm{eff}}$ so that Ψ approaches zero more rapidly. Results shown in Fig. 8 are even more interesting. The linear-absorption approximation predicts defocusing of a laser beam of any value of the incident axial intensity I_0 when $S=2$. Consideration of nonlinearity in the absorption, however, predicts focusing of the beam if I_0 fulfills certain conditions. These effects are obvious from the variations of P_a, Q_a, P_2, and Q_2 depicted in Fig. 1.

We thus conclude from the present investigation that the effect of nonlinear absorption on self-focusing of a laser beam in a plasma is very much significant. A laser beam can become focused even when the linear-absorption approximation[1,6] would predict defocusing of the beam.

ACKNOWLEDGMENTS

The authors are very grateful to Dr. D.P. Tewari, Dr. S.C. Kaushik, and D. Subba Rao for stimulating discussions and encouragement during the present investigation.

[1]M.S. Sodha, A.K. Ghatak, and V.K. Tripathi, in *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1976), Vol. XIII, p. 169.

[2]M.S. Sodha and V.K. Tripathi, in *Laser Interaction and Related Plasma Phenomena*, edited by H.J. Schwarz and H. Hora (Plenum, New York, 1977), Vol. IV; M.S. Sodha and V.K. Tripathi, Phys. Rev. A 16, 2101 (1977).

[3]S.A. Akhmanov, R.V. Khokhlov, and A.P. Sukhorukov, in *Laser Handbook*, edited by F.T. Arecchi and E.O. Schulz-Du Bois (North-Holland, Amsterdam, 1972), Vol. 2, p. 1151.

[4]O. Svelto, in *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1974), Vol. XII, p. 1; Y.R. Shen, Prog. Quantum Electron. **4**, 1 (1975).

[5]H. Hora, J. Opt. Soc. Am. **65**, 882 (1975).

[6]C.J.K. Virmani, R.G. Gupta, and V.K. Tripathi, Optik (Stuttgart) **40**, 431 (1974).

[7]S.Y. Yuen, Appl. Phys. Lett. **30**, 223 (1977).

[8]M.S. Sodha and A.K. Ghatak, *Inhomogeneous Optical Waveguides* (Plenum, New York, 1977); D. Marcuse, *Light Transmission Optics* (Van Nostrand—Reinhold, Princeton, 1972).

[9]V.L. Ginzburg, *Electromagnetic Wave Propagation in Plasmas* (Pergamon, New York, 1970).