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Effect of self-focusing on scattering of a laser beam in a collisional plasma

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Abstract. The temporal growth rates of stimulated Raman and Brillouin scattering of a self-focused laser beam in a collisional plasma have been evaluated. The calculations predict a considerable spatial nonuniformity in the scattering because of the temperature and density gradients induced in the plasma on account of the nonuniformity of the pump laser beam.

1. Introduction

Raman and Brillouin scattering of laser beams (Kaiser and Maier 1972) occurs in a variety of media: e.g. in optical waveguides (Maier 1976), laboratory plasmas (Hellwarth 1976) and fusion plasmas (Brueckner and Jorna 1974). The reported investigations suffer from the limitation that they assume the laser beam to be uniform. In fact, a laser beam is initially radially nonuniform and therefore also attains longitudinal nonuniformity on account of the phenomenon of self-focusing (Akhmanov *et al* 1972, Svelto 1974, Sodha *et al* 1976a).

The present paper is an attempt to understand the effect of self-focusing of a laser beam on temporal growth rates of stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) of the laser beam in an inherently homogeneous unmagnetised collisional plasma; in a collisional plasma, the nonuniform heating and consequent redistribution of the plasma, on account of a radially nonuniform laser beam, is mainly determined by collisions of electrons with heavier particles. As a first approximation, we ignore the effect of SRS and SBS on self-focusing of the beam; hence the results obtained herein are not applicable to very high powers. The analysis is based on the parametric, WKB, paraxial and local field approximations. In §2, the relevant results from the self-focusing theory are stated. In §3, the standard expressions for the temporal growth rates of SRS and SBS are stated. In §4, spatial dependence of these growth rates on account of self-focusing is discussed. In §5, typical numerical results and discussion are presented.

The present investigation predicts a considerable spatial nonuniformity in the temporal growth rates of SRS and SBS. The spatial dependence of these growth rates and the propagation-distance dependence of the beamwidth parameter of the pump laser beam are correlated. But this correlation is not followed evenly, because due to carrier heating as well as redistribution, the growth rates do not necessarily increase with the pump intensity.

2. Self-focusing of the pump

Consider a laser beam propagating along the z axis in a formerly homogeneous unmagnetised collisional plasma. The electric vector of the laser beam at $z=0$ may be expressed as

$$E(r, z=0, t) = \hat{E}E_0 \exp(-r^2/2r_0^2 - i\omega t) \tag{1}$$

where $\hat{E}$ is the unit vector representing the polarisation of E and r_0 is the initial mean beam-width radius.

The time scale of temporal variation of the incident pump intensity E_0^2 will be assumed to be much larger than the reciprocal of electron collision frequency ν^{-1}. The electron temperature then increases (Sodha *et al* 1976a) to such an extent that the electron-ion redistribution due to ponderomotive force may be neglected as compared to that due to heating and that the redistribution process follows a steady-state model. assuming the pump frequency ω to be much larger than the electron collision frequency ν and assuming the usual parametric approximation (Brueckner and Jorna 1974), any depletion of the pump intensity may be neglected. In the present investigation, the plasma will be assumed to contain only electrons and singly charged positive ions of one type.

Under these conditions, the effective dielectric constant of the plasma becomes intensity-dependent in the form (Sodha *et al* 1976a)

$$\epsilon = \epsilon_0 + \Phi \tag{2}$$

where

$$\epsilon_0 = 1 - \omega_{p0}^2/\omega^2 \tag{3a}$$

$$\Phi = (\omega_{p0}^2/\omega^2)(1+\alpha|E|^2)^{-1}\alpha|E|^2 \tag{3b}$$

$$\omega_{p0}^2 = 4\pi N_0 e^2/m \tag{3c}$$

$$\alpha = e^2 M/12 m^2\omega^2 k_B T_0 \tag{3d}$$

N_0 is the electron (or ion) concentration in the absence of the pump, m is the electron mass, M is the ion mass, k_B is the Boltzmann constant and T_0 is the initial electron temperature. Note that $\Phi(|E|^2=0)=0$, as expected from the boundary condition employed.

The wave equation for E, when solved under the WKB and paraxial approximations, gives the following solution valid in the nonresonant ($\epsilon_0 \neq 0$) paraxial ($r \ll r_0$) region (Akhmanov *et al* 1972, Svelto 1974, Sodha *et al* 1976a)

$$E(r, z, t) = \hat{E}E_0 f^{-1} \exp[-r^2/2r_0^2 f^2 + iK(S+z) - i\omega t] \tag{4}$$

where

$$K = \epsilon_0^{1/2}\omega/c \tag{5}$$

is the linear wavenumber and $S(r, z)$, which may be expressed in terms of $f(z)$, is (z/K) times the nonlinear part of the wavenumber. The beamwidth parameter $f(z)$ is governed by the equation (Sodha *et al* 1976a)

$$f^3 \frac{d^2 f}{dz^2} = \frac{1}{K^2 r_0^4} - \frac{\omega_{p0}^2/\omega^2}{2\epsilon_0 r_0^2} \frac{\alpha E_0^2}{(1+\alpha E_0^2/2f^2)^2} \tag{6}$$

along with the boundary conditions

$$f(z=0) = 1 \quad \text{and} \quad (df/dz)_{z=0} = 0. \tag{7}$$

Equation (6) can be solved numerically by the Runge-Kutta method (Scarborough

1966); the accuracy of the solution depends upon the choice of the step-length Δz. Sodha and Tripathi (1977) and Sodha *et al* (1978b) have investigated the behaviour of f on the basis of (6). They have shown that there exist two values of E_0^2, namely

$$E_{t\mp}^2 = [\beta - 1 \mp (\beta^2 - 2\beta)^{1/2}](2/\alpha) \tag{8}$$

with

$$\beta = K^2 r_0^2 \omega_{p0}^2 / 2\epsilon_0 \omega^2 \tag{9}$$

for which the beam gets self-trapped whereby $f(z) = 1$. For $E_{t-}^2 < E_0^2 < E_{t+}^2$, the beam gets focused whereby $f(z)$ oscillates between 1 and f_{min} (<1). For $E_0^2 < E_{t-}^2$ or $E_0^2 > E_{t+}^2$, the beam gets defocused whereby $f(z)$ oscillates between 1 and f_{max} (>1).

3. Scattering of a uniform pump

3.1. Stimulated Raman scattering (SRS)

For pump frequency $\omega >$ twice the plasma frequency $2\omega_p$, a pump photon may be scattered as another photon polarised along some direction $\hat{E}_s$ by exciting a plasmon (quantum of electron plasma oscillations) of arbitrary wavenumber K_e and the corresponding frequency

$$\omega_e = \omega_p (1 + 3\, K_e^2 \lambda_D^2 / 2)^{1/2} \tag{10}$$

where the Debye length

$$\lambda_D = (k_B T_e / m\omega_p^2)^{1/2} \tag{11a}$$

$$T_e = T_0 (1 + 2\alpha |E|^2) \tag{11b}$$

$$\omega_p^2 = \omega_{p0}^2 (1 + \alpha |E|^2)^{-1}. \tag{11c}$$

In a cold homogeneous unmagnetised plasma, the temporal growth rate of SRS of a uniform pump laser beam is given by (Liu and Kaw 1976)

$$\gamma(\text{SRS}) = \{[1 + (K - K_e)^2 c^2 / \omega_p^2]^{-1/2} [E.\hat{E}_s K_e c / m\omega]^2 + (\Gamma_2 - \Gamma_p)^2 / 4\}^{1/2} - (\Gamma_2 + \Gamma_p)/2 \tag{12}$$

where

$$\Gamma_2 = \nu_e \omega_p^2 / 2\omega^2 \tag{13}$$

is the collisional damping rate of the scattered electromagnetic wave, and

$$\Gamma_p = (\pi^{1/2} \omega_p / 2K_e^3 \lambda_D^3) \exp[-3/4 - K_e^2 \lambda_D^2 / 2] + \nu_{ec} \tag{14}$$

is the Landau plus collisional damping rate of electron plasma oscillations (Akhiezer *et al* 1975).

3.2. Stimulated Brillouin scattering (SBS)

For $\omega > \omega_p$, a pump photon may be scattered as another photon polarised along some direction $\hat{E}_s$ by exciting a phonon (quantum of ion plasma oscillations) of arbitrary wavenumber K_i and the corresponding frequency

$$\omega_i = \omega_p (m/M)^{1/2} (1 + 1/K_i^2 \lambda_D^2)^{-1/2}. \tag{15}$$

In a cold homogeneous unmagnetised plasma, the temporal growth rate of SBS of a uniform pump laser beam is given by (Liu and Kaw 1976)

$$\gamma(\text{SBS}) = [\{(\omega_p/\omega)(m/M)^{1/2}(1+1/K_t^2\lambda_D^2)^{1/2}[E.\hat{E}_s K_t e/m\omega(1+K_t^2\lambda_D^2)]\}^2 + (\Gamma_2-\Gamma_a)^2/4]^{1/2} - (\Gamma_2+\Gamma_a)/2 \tag{16}$$

where

$$\Gamma_a = (T_e/T_0)^{3/2}\exp(-3/2 - T_e/2T_0) + K_t c_s(\pi m/8M)^{1/2} + \nu_{ie} \tag{17}$$

is the Landau plus collisional damping rate of ion plasma oscillations, and

$$c_s = (k_B T_0/M)^{1/2} \tag{18}$$

is the ion-acoustic speed (Akhiezer *et al* 1975).

4. Scattering of a self-focused pump

The expressions for temporal growth rates of SRS and SBS in the case of an inhomogeneous plasma or a nonuniform pump laser beam are slightly different (Nishikawa and Liu 1976) from that given by (12) and (16). Sodha *et al* (1978a) have, however, shown that when the mean beamwidth and self-focusing length of the pump are assumed to be much longer than the wavelengths involved in the scattering, the foregoing expressions along with the local field parameters are quite justified.

For illustration, the following parameters have been chosen:

$N_0 = 1\times10^{18}$ cm^{-3} for SRS and 5×10^{18} cm^{-3} for SBS

$M/m = 2000$

$k_B T_0 = 1.38\times10^{-9}$ erg

$\nu_e = \nu_{ec} = \nu_{ie} \ll \omega$

$\omega = 1.84\times10^{14}$ rad s^{-1}

$r_0 = 0.005$ cm

$\hat{E}_s = \hat{E}$

$K_e = K_t = 2\,100$ cm^{-1}.

These parameters yield

$E_{t-}^2/(10^9 \text{ erg cm}^{-3}) = 24.84$ for SRS and 4.880 for SBS;

$E_{t+}^2/(10^{11} \text{ erg cm}^{-3}) = 1.913$ for SRS and 9.742 for SBS.

The incident pump intensity E_0^2 will be allowed to take the following five different values in each case:

(1) $E_0^2 = E_{t-}^2/2$

(2) $E_0^2 = E_{t-}^2$

(3) $E_0^2 = (E_{t-}^2 + E_{t+}^2)/2$

(4) $E_0^2 = E_{t+}^2$

(5) $E_0^2 = 2E_{t+}^2$.

These parameters have been so chosen that the modifications required in the expressions for the temporal growth rates, on account of the laser beam induced inhomogeneity, are negligible so that the use of (12) and (16) along with the local field parameters is justified. The values of E_0^2 have been chosen as above with the aim to illustrate all the manifestations of the phenomenon of self-focusing (Sodha and Tripathi 1977, Sodha *et al* 1978b).

The beamwidth parameters f versus z for SRS and SBS are plotted in figure 1 (*a* and *b*). The temporal growth rates γ_0 at $r=0$ versus z for SRS and SBS are plotted in figure 2 (*a* and *b*). The temporal growth rates γ_1 at $r=0{\cdot}01$ cm versus z for SRS and SBS are plotted in figure 3 (*a* and *b*). In view of the paraxial approximation employed, the graphs of figure 3 (*a* and *b*) can be claimed to be correct only approximately.

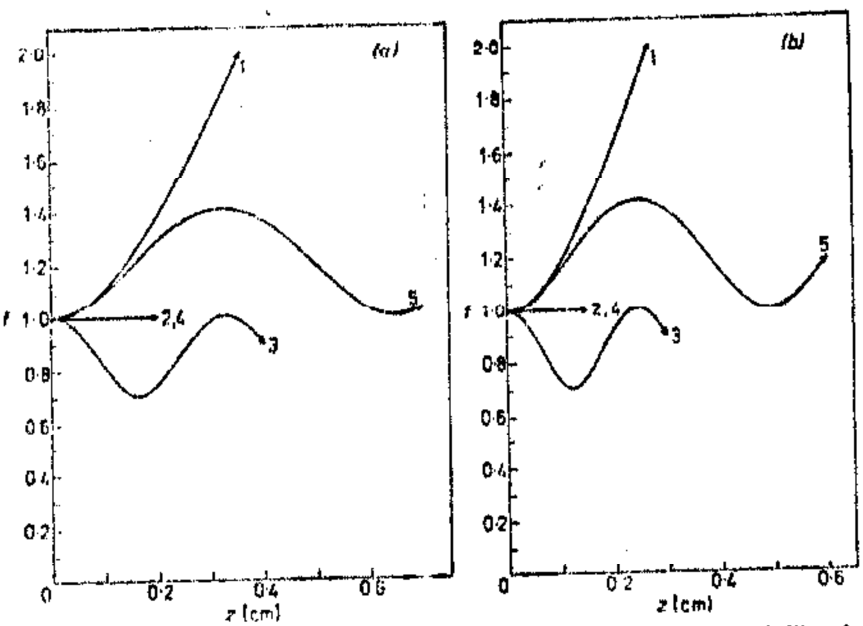

Figure 1. Beam width parameter f versus z for (*a*) SRS; (*b*) SBS. Numbers labelling the curves denote the incident pump intensity as follows: 1, $E_0^2 = E_{t-}^2/2$; 2, $E_0^2 = E_{t-}^2$; 3, $E_0^2 = (E_{t-}^2 + E_{t+}^2)/2$; 4, $E_0^2 = E_{t+}^2$; 5, $E_0^2 = 2E_{t+}^2$.

5. Discussion

Some of the general observations from figures 1, 2 and 3 are as follows. For $E_0^2 = E_{t-}^2$ or E_{t+}^2, $f(z)=1$ and $\gamma_1(z)=\gamma_1(0)<\gamma_0(z)=\gamma_0(0)$ in such a way that γ (for $E_0^2=E_{t-}^2$) < γ (for $E_0^2=E_{t+}^2$). For $E_{t-}^2 < E_0^2 < E_{t+}^2$, f and γ_1 oscillate between their values at $z=0$ and certain minimum values, whereas γ_0 oscillates between its value at $z=0$ and a certain maximum value. For $E_0^2 < E_{t-}^2$ or $E_0^2 > E_{t+}^2$, f and γ_1 oscillate between their values at $z=0$ and certain maximum values, whereas γ_0 oscillates between its value at $z=0$ and a certain minimum value. The conclusion that γ_1 has a similar variation with z as f, whereas γ_0 behaves oppositely, can be explained as follows. The temporal growth rate increases with the pump intensity and the carrier density, which, in turn, decrease and increase respectively with f. In the paraxial region, the effect of change of the pump

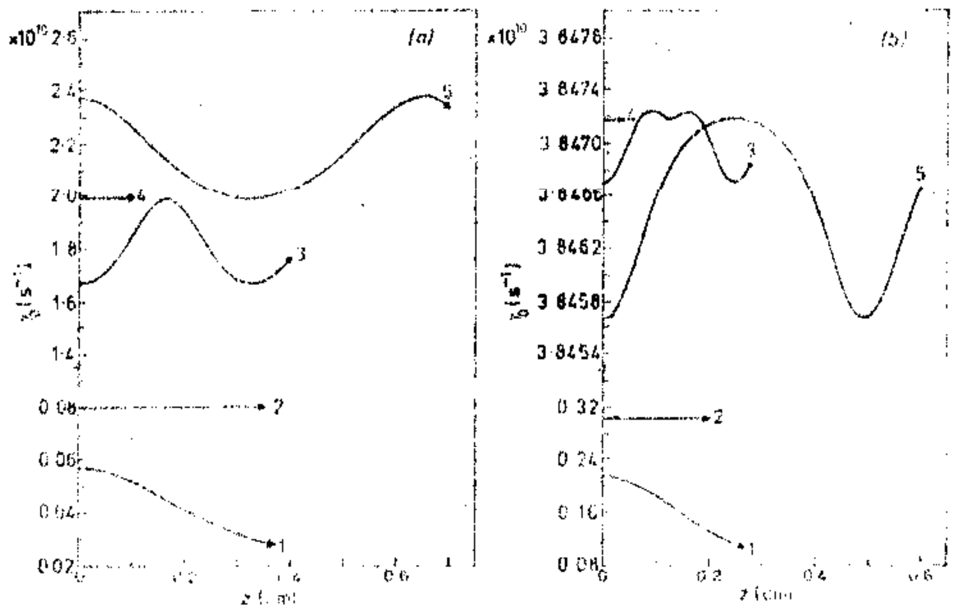

Figure 2. Temporal growth rate γ_0 at $r=0$ versus z for (*a*) SRS, (*b*) SBS. Numbers labelling the curve denote the incident pump intensity as defined in figure 1.

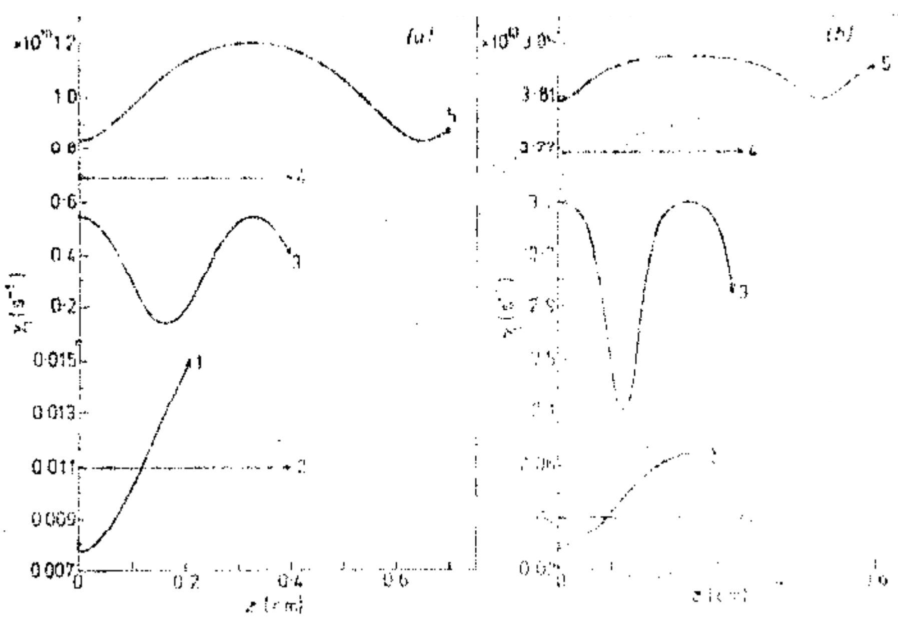

Figure 3. Temporal growth rate γ_1 at $r=0{\cdot}01$ cm versus z for (*a*) SRS, (*b*) SBS. Numbers labelling the curves denote the incident pump intensity as defined in figure 1.

intensity dominates and hence γ_0 decreases with f. In the off-axial region, on the other hand, the effect of change of the carrier density dominates and hence γ_1 increases with f.

However, the forementioned correlation is not followed evenly. In figure 2(*b*), the graph for $E_0^2=(E_{l-}^2+E_{l+}^2)/2$ has two peaks rather than a single peak within one oscillatory unit of length. In the same figure, the position of the graph for $E_0^2=2E_{l+}^2$

is inverted as compared to its position expected in view of the forementioned correlation. This odd behaviour does not appear in a collisionless plasma (Sodha *et al* 1978a) in which carrier heating does not take place. We therefore conclude that the increase in the electron temperature, induced by the laser beam, has the effect of complicating the functional dependence of the temporal growth rates, particularly that of γ (SBS).

The temporal growth rates of SRS and SBS oscillate with the propagation distance z and moreover decrease or increase with the radial coordinate r. To have a numerical appreciation of this spatial nonuniformity, the following particular cases may be useful. For $E_0^2=(E_{L-}^2+E_{L+}^2)/2$, the ratios $\gamma_{0max}/\gamma_{0(z=0)}$, $\gamma_{1(z=0)}/\gamma_{0(z=0)}$, $\gamma_{1min}/\gamma_{0(z=0)}$ are respectively 1·192, 0·32, 0·08 for SRS and 1·001, 0·96, 0·55 for SBS. These ratios are appreciably different from unity and thereby indicate appreciable nonuniformity in the temporal growth rates of SRS and SBS.

The time scale of temporal variation of the incident pump intensity E_0^2 has been assumed, in the present investigation, to be much larger than reciprocal of the electron collision frequency ν^{-1}. When it is not so, the carrier heating does not take place and redistribution takes place due to a ponderomotive force mechanism rather than a collisional heating mechanism. Sodha *et al* (1978a) have studied the temporal growth rates of SRS, SBS, two-plasmon decay instability and plasmon–phonon decay instability in such a 'collisionless' plasma. They have, however, considered only a limited range of values of E_0^2 so that not all of the manifestations of the phenomenon of self-focusing (Sodha *et al* 1978b) are examined. Because of the absence of carrier heating, their results do not show any odd behaviour observed in the present investigation. Their results (Sodha *et al* 1978b) as well as results of the present investigation show that temporal growth rates of SRS and SBS are enhanced if $E_{L-}^2 < E_0^2 < E_{L+}^2$. This result is consistent with the Cotter *et al* (1975) result that SRS is enhanced by external focusing of the pump and the Sodha *et al* (1976b) result that SRS is enhanced by self-focusing of the pump.

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