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Self-focusing of a laser beam in a magnetoplasma

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Abstract. This paper presents an investigation of the self-focusing of an elliptically gaussian laser beam propagating along the static magnetic field in a non-linearly absorbing magnetoplasma. The electron concentration and temperature and the static magnetic field are allowed to be non-uniform. The direct coupling as well as the indirect coupling between the right- and left-handedly polarized modes has been taken into account. Coupled equations for the beamwidth parameters have been derived by using the Akhmanov approach. It is seen that axial symmetry of the intensity distributions of the two modes is not preserved in the presence of the direct coupling. The variations of the beam-width parameters become considerably aperiodic on account of the indirect coupling and non-uniformity in the plasma parameters. Though the investigation is explicitly related to a magnetoplasma, the analysis presented herein can be easily extended for some other anisotropic media.

1. Introduction

First hypothesized to explain an observation of anomalous laser scattering [1, 2], the phenomenon of laser self-focusing [1–6] has now acquired a distinct status in the realm of non-linear optics [6]. Laser self-focusing has now been observed and investigated in almost all kinds of material media ; recently it has drawn the attention of plasma physicists interested in the ionospheric modulation [7] or heating of fusion plasmas [8] with laser beams. Except for the derivation of the dielectric constant, theoretical analyses of the phenomenon are common for all media ; thus, theoretical investigations of the laser self-focusing carried out by non-plasma physicists are of equal interest to plasma physicists and vice versa. It is to be further noted that several new concepts and mathematical techniques have emerged in the course of theoretical treatments of the phenomenon [3].

The analysis of laser self-focusing becomes particularly complicated when the medium is anisotropic [1, 5] as can be realized, for example, when one considers the presence of a static magnetic field in a plasma [9, 10]. In order to make the problem of laser self-focusing in a magnetoplasma solvable, earlier investigations [11–16] made the following simplifying assumptions regarding the static magnetic field : (i) the static magnetic field is directed without having any toroidal component present ; (ii) it is uniform ; (iii) the laser beam propagates along or across the static magnetic field ; (iv) one of the two modes (right- and left-handedly polarized modes in the case of propagation along the static magnetic field or ordinary and extraordinary modes in the case of propagation across it) is absent ; (iv′) when both the modes are present, their direct coupling is negligible. (We shall, for convenience, call the coupling between the two modes direct if it arises on account of the presence of the static magnetic field and indirect if it arises on account of the intensity dependence of the dielectric tensor.)

In this paper, we have investigated the self-focusing of an elliptically gaussian laser beam propagating along the static magnetic field in a non-linearly absorbing plasma. The electron concentration and temperature (even in the absence of the laser beam) and the static magnetic field are allowed to be non-uniform. Both the right- and left-handedly polarized modes are allowed to be present; their direct coupling as well as indirect coupling has been taken into account. In § 2, we have presented the expressions for the dielectric-tensor components $\epsilon_{\pm}$ for the two modes when their intensity dependence arises on account of the relativistic mass variation [17], ponderomotive force [18] or collisional heating [19] of electrons. In § 3, we have solved the wave equation for the electric field in terms of the 'modified' beamwidth parameters which are governed by second-order coupled differential equations; we have adopted a slightly modified version [20] of the Akhmanov approach [1–6] which is based on the WKB and paraxial approximations. In § 4, we have presented numerical results along with a discussion. The entire analysis (in § 2 and § 3) has been so presented that it can be easily extended for some other anisotropic media.

We infer from the present investigation that the direct coupling between the two modes destroys the axial symmetry of their intensity distributions as the two modes propagate inside the plasma. On account of the indirect coupling between the two modes and non-uniformity in the plasma parameters, the criteria for focusing/defocusing of the two modes are altered and the variations of the beamwidth parameters become considerably aperiodic.

2. Dielectric tensor

The nine components of the dielectric tensor $\boldsymbol{\epsilon}$ of a plasma magnetized along the direction $\hat{z}$ of propagation of the laser beam, at frequency ω much greater than the non-relativistic cyclotron frequency (eB_0/m_0c) as well as the quiescent electron collision frequency ν_0, are given by the following expressions [5, 20, 21].

$$\epsilon_{\parallel}=\epsilon_{zz}=1-w_{\parallel}P-iv_{\parallel}Q, \tag{2.1}$$

$$\epsilon_{\pm}=\epsilon_{xx}\mp i\epsilon_{xy}=\epsilon_{yy}\pm i\epsilon_{yx}=1-w_{\pm}P-iv_{\pm}Q, \tag{2.2}$$

$$\epsilon_{xz}=\epsilon_{zx}=\epsilon_{yz}=\epsilon_{zy}=0, \tag{2.3}$$

where

$$w_{\parallel}=4\pi N_0e^2/m_0\omega^2, \tag{2.4}$$

$$v_{\parallel}=w_{\parallel}\nu_0/\omega, \tag{2.6}$$

$$w_{\pm}=w_{\parallel}/(1\mp\gamma), \tag{2.6}$$

$$v_{\pm}=v_{\parallel}/(1\mp\gamma), \tag{2.7}$$

$-e$ is the electron charge, m_0 is the electron rest mass, N_0 is the electron concentration in the absence of the field, ν_0 is the electron collision frequency in the absence of the electromagnetic field,

$$\gamma=eB_0/m_0c\omega \tag{2.8}$$

is the ratio of the non-relativistic cyclotron frequency to the laser frequency, $B_0\hat{z}$ is the static magnetic field,

$$P = Nm_0/N_0m \tag{2.9}$$

represents the electromagnetic field induced variation in the ratio N/m of the electron concentration to the electron mass,

$$Q = Nm_0\nu/N_0m\nu_0 = (N/N_0)^b(m_0/m)^d(T/T_0)^{s/2} \tag{2.10}$$

represents the electromagnetic field induced variation in the product $N\nu/m$ of N/m and the electron collision frequency ν, T is the electron temperature, T_0 is the electron temperature in the absence of the electromagnetic field,

$$b = 2 \quad \text{or} \quad 1 \tag{2.11}$$

depending upon whether the electron collisions are with ions or neutrals,

$$d = 1 - s/2, \tag{2.12}$$

and

$$s = -3 \quad \text{or} \quad 1 \quad \text{or} \quad 2 \tag{2.13}$$

depending upon whether the electron collisions are with ions or non-diatomic molecules or diatomic molecules [21].

It is convenient to use the dimensionless intensity [20]

$$I = I_+ + I_-, \tag{2.14}$$

$$I_\pm = \beta\Gamma_\pm E_\pm^* \cdot E_\pm, \tag{2.15}$$

where the normalization constants $\beta\Gamma_\pm$ depend on the mechanism of non-linearity under consideration, $\Gamma_\pm$ are such that $\Gamma_\pm = 1$ in the absence of the static magnetic field, and

$$E_\pm = E_x \pm iE_y \tag{2.16}$$

are the electric fields [5] corresponding to the right (−) and left (+) handedly polarized modes. Then it can be shown that

$$\beta = e^2/m_0^2 c^2 \omega^2, \tag{2.17 a}$$

$$\Gamma_\pm = (1 \mp \gamma)^{-2}, \tag{2.18 a}$$

$$P = (1 + I)^{-1/2}, \tag{2.19 a}$$

$$Q = (1 + I)^{-d/2}, \tag{2.20 a}$$

in the case of relativistic mass variation mechanism [17] ;

$$\beta = e^2/4m_0k_B T_0\omega^2, \tag{2.17 b}$$

$$\Gamma_\pm = (1 - \gamma/2)(1 \mp \gamma)^{-2}, \tag{2.18 b}$$

$$P = \exp(-I), \tag{2.19 b}$$

$$Q = \exp(-bI), \tag{2.20 b}$$

in the case of ponderomotive force mechanism [18] ; and

$$\beta = Me^2/6m_0^2 k_B T_0\omega^2, \tag{2.17 c}$$

$$\Gamma_\pm = (1 \mp \gamma)^{-2}, \tag{2.18 c}$$

$$P = (1 + I)^{s/2-1}, \tag{2.19 c}$$

$$Q = (1 + I)^{b(s/2-1)}(1 + 2I)^{s/2}, \tag{2.20 c}$$

in the case of collisional heating mechanism [19]. Here M is the ion/neutral rest mass and k_B is the Boltzmann constant. It should be mentioned here that the relativistic variation of the cyclotron frequency (eB_0/mc) and the presence of longitudinal electric field E_z have been ignored in writing equations (2.2) and (2.14) respectively.

We shall assume that in the paraxial region, $w_{\parallel}$, $v_{\parallel}$, γ and $\beta\Gamma_{\pm}$ vary radially only in powers of χ^2 and η^2;

$$\chi = x/r_0 \quad \text{and} \quad \eta = y/r_0 \tag{2.21}$$

are the transverse cartesian coordinates normalized by an arbitrary length r_0. We shall express the dimensional intensities of the two modes as

$$E_{\pm}^* \,.\, E_{\pm} = E_{\pm 0}^2 \exp\,[h_{\pm ar} - \sum_L h_{\pm Lr}\phi_L], \tag{2.22}$$

where

$$\phi_L = \chi^2, \quad \eta^2 \quad \text{or} \quad \chi\eta \tag{2.23}$$

depending upon whether $L=1$, 2 or 3. We may then expand the dielectric tensor components $\epsilon_{\pm}$, in the paraxial region, as follows [20].

$$\epsilon_{\pm} = (\epsilon_{\pm ar} - i\epsilon_{\pm ai}) - \sum_L (\epsilon_{\pm Lr} + i\epsilon_{\pm Li})\phi_L, \tag{2.24}$$

where

$$\epsilon_{\pm ar} = 1 - w_{\pm a}P_a, \tag{2.25}$$

$$\epsilon_{\pm ai} = v_{\pm a}Q_a, \tag{2.26}$$

$$\epsilon_{\pm Lr} = w_{\pm L}P_a + w_{\pm a}R(-dP_a/dI_a), \tag{2.27}$$

$$\epsilon_{\pm Li} = v_{\pm L}Q_a + v_{\pm a}R(-dQ_a/dI_a); \tag{2.28}$$

$$w_{\pm a} = w_{\pm}(\chi = \eta = 0), \tag{2.29}$$

$$v_{\pm a} = v_{\pm}(\chi = \eta = 0), \tag{2.30}$$

$$w_{\pm L} = (\partial w_{\pm}/\partial \phi_L)_{(\chi = \eta = 0)}, \tag{2.31}$$

$$v_{\pm L} = (\partial v_{\pm}/\partial \phi_L)_{(\chi = \eta = 0)}, \tag{2.32}$$

$$P_a = P(\chi = \eta = 0) = P(I = I_a), \tag{2.33}$$

$$Q_a = Q(\chi = \eta = 0) = Q(I = I_a), \tag{2.34}$$

$$\begin{aligned} R &= -(\partial I/\partial \phi_L)_{(\chi = \eta = 0)} \\ &= \{[h_{+Lr} - \partial \ln(\beta\Gamma_+)/\partial \phi_L]I_+ + [h_{-Lr} - \partial \ln(\beta\Gamma_-)/\partial \phi_L]I_-\}_{(\chi = \eta = 0)}, \end{aligned} \tag{2.35}$$

$$I_a = I(\chi = \eta = 0). \tag{2.36}$$

The presence of $w_{\pm L}$ in equation (2.27), $v_{\pm L}$ in equation (2.28) and $\partial \ln(\beta\Gamma_{\pm})/\partial\phi_L$ in equation (2.35) makes the present analysis applicable even when the electron concentration and temperature (in the absence of the laser beam) and the static magnetic field are radially non-uniform [9]. In writing equations (2.22) and (2.24), we have assumed that the derivatives $\partial h_{\pm}/\partial\chi$, $\partial h_{\pm}/\partial\eta$, $\partial w_{\pm}/\partial\chi$, $\partial w_{\pm}/\partial\eta$, $\partial v_{\pm}/\partial\chi$, $\partial v_{\pm}/\partial\eta$, $\partial\beta\Gamma_{\pm}/\partial\chi$, $\partial\beta\Gamma_{\pm}/\partial\eta$ are negligibly small, if at all present.

3. Beam propagation

When $k_0^2|\epsilon| \gg |\nabla^2 \epsilon|$, the electric fields corresponding to the right (−) and left (+) handedly polarized modes are governed by the scalar wave equations [14–16]

$$\left[\frac{\partial^2}{\partial \xi^2} + q\sigma_\pm \left(\frac{\partial^2}{\partial \chi^2} + \frac{\partial^2}{\partial \eta^2}\right) + q^2 \epsilon_\pm\right] E_\pm = q(1-\sigma_\mp)\left(\frac{\partial}{\partial \chi} \pm i\frac{\partial}{\partial \eta}\right)^2 E_\mp, \quad (3.1)$$

where we have used the notations

$$\xi = z/k_0 r_0^2, \quad (3.2)$$

$$q = k_0^2 r_0^2, \quad (3.3)$$

$$\sigma_\pm \simeq 1/2 + \epsilon_{\pm ar}/2(1 - w_\parallel P_a), \quad (3.4)$$

$$k_0 = \omega/c. \quad (3.5)$$

Note that the two modes are coupled directly on account of the presence of the right-hand-side term in equation (3.1) and indirectly on account of the intensity dependence of the dielectric tensor components $\epsilon_\pm$. Earlier investigations [11–16] have neglected the direct coupling, and hence they are applicable only when $\gamma \simeq 0$ or $\xi \simeq 0$. In the present investigation, we shall retain the direct-coupling term as far as possible.

As a boundary condition, we take the electric field of the laser beam at $\xi = 0$ to be given by

$$E_\pm(\xi = 0) = E_{\pm 0} \exp\left[i\omega t - h_{\pm 1r0}\chi^2/2 - h_{\pm 2r0}\eta^2/2\right]. \quad (3.6)$$

Without loss of generality, we may consider the transverse electric field $\mathbf{E}_\perp(t = \xi = 0)$ at $t = \xi = 0$ to be along the x-axis so that $E_{\pm 0}$ are real. If the beam diverges/converges very slowly, then we may express the solution of equation (3.1) in the paraxial region in the form [14–16, 20]

$$E_\pm = E_{\pm 0} \exp\left[i\omega t - iq\int k_\pm\, d\xi + (g_{\pm r} + ig_{\pm i})/2 - \sum_L (h_{\pm Lr} + ih_{\pm Li})\phi_L/2\right], \quad (3.7)$$

where

$$k_\pm = \epsilon_{\pm ar}^{1/2} \quad (3.8)$$

and $g_\pm$ and $h_{\pm L}$ vary with ξ but not with χ or η. We have assumed $|\epsilon_{\pm ai}| \ll |\epsilon_{\pm ar}|$ in writing the above expression for k. Substituting for $E_\pm$ from the expression (3.7) into equation (3.1), and then equating the real and imaginary coefficients of $\phi_{\pm L}^0$ and $\phi_{\pm L}$ on both sides of the resulting equation, we obtain 16 second-order coupled differential equations for $g_{\pm r}$, $g_{\pm i}$, $h_{\pm Lr}$ and $h_{\pm Li}$. (By allowing $\phi_{\pm L}$ to be $\phi_{\pm 4} = \chi$ and $\phi_{\pm 5} = \eta$ and thereby increasing the number of coupled differential equations from 16 to 24, we can justify our assumption that $h_{\pm 4r}$, $h_{\pm 4i}$, $h_{\pm 5r}$ and $h_{\pm 5i}$ are negligibly small.) In the WKB approximation, we neglect the second-order terms in these 16 equations and obtain

$$k_\pm \frac{dg_{\pm i}}{d\xi} - \sigma_\pm \sum_l h_{\pm lr} - q\epsilon_{\pm 0ar} = 0, \quad (3.9)$$

$$k_\pm \frac{dg_{\pm r}}{d\xi} + \sigma_\pm \sum_l h_{\pm li} + \frac{dk_\pm}{d\xi} + q\epsilon_{\pm aI} = 0, \quad (3.10)$$

$$k_{\pm}\frac{dh_{\pm li}}{d\xi}-\sigma_{\pm}(h_{\pm lr}^{2}-h_{\pm li}^{2})+q\epsilon_{\pm lR}=0, \tag{3.11}$$

$$k_{\pm}\frac{dh_{\pm lr}}{d\xi}+2\sigma_{\pm}h_{\pm lr}h_{\pm li}-q\epsilon_{\pm lI}=0, \tag{3.12}$$

$$k_{\pm}\frac{dh_{\pm 3i}}{d\xi}-\sigma_{\pm}\sum_{l}h_{\pm lr}h_{\pm 3r}+\sigma_{\pm}\sum_{l}h_{\pm li}h_{\pm 3i}+q\epsilon_{\pm 3R}=0, \tag{3.13}$$

$$k_{\pm}\frac{dh_{\pm 3r}}{d\xi}+\sigma_{\pm}\sum_{l}h_{\pm lr}h_{\pm 3i}+\sigma_{\pm}\sum_{l}h_{\pm li}h_{\pm 3r}-q\epsilon_{\pm 3I}=0, \tag{3.14}$$

where $l=1$ and 2, and

$$\epsilon_{\pm adr}=[\mathrm{Re}\ \epsilon_{\pm d}]_{(\chi=\eta=0)}, \tag{3.15}$$

$$\epsilon_{\pm aI}=\epsilon_{\pm ai}+[\mathrm{Im}\ \epsilon_{\pm d}]_{(\chi=\eta=0)}, \tag{3.16}$$

$$\epsilon_{\pm LR}=\epsilon_{\pm Lr}+[(\partial/\partial\phi_{L})\ \mathrm{Re}\ \epsilon_{\pm d}]_{(\chi=\eta=0)}, \tag{3.17}$$

$$\epsilon_{\pm LI}=\epsilon_{\pm Li}+[(\partial/\partial\phi_{L})\ \mathrm{Im}\ \epsilon_{\pm d}]_{(\chi=\eta=0)}, \tag{3.18}$$

$$\epsilon_{\pm d}=E_{\mp}^{-1}\,q^{-1}(1-\sigma_{\mp})(\partial/\partial\chi\pm i\partial/\partial\eta)^{2}\,E_{\mp}. \tag{3.19}$$

Equations (3.12) and (3.14) lead to

$$h_{\pm li}=(q\epsilon_{\pm lI}-k_{\pm}dh_{\pm lr}/d\xi)/2\sigma_{\pm}h_{\pm lr}, \tag{3.20}$$

$$h_{\pm 3i}=(q\epsilon_{\pm 3I}-k_{\pm}\ dh_{\pm 3r}/d\xi-\sigma_{\pm}\sum_{l}h_{\pm li}h_{\pm 3r})/\sigma_{\pm}\sum_{l}h_{\pm lr}. \tag{3.21}$$

Substituting for $h_{\pm li}$ and $h_{\pm 3i}$ from the expressions (3.20) and (3.21) into equations (3.11) and (3.13), we then obtain

$$\frac{d^{2}h_{\pm lr}}{d\xi^{2}}-\frac{3}{2h_{\pm lr}}\left(\frac{dh_{\pm lr}}{d\xi}\right)^{2}+\left(\frac{1}{k_{\pm}}\frac{dk_{\pm}}{d\xi}+\frac{2q\epsilon_{\pm lI}}{h_{\pm lr}k_{\pm}}\right)\frac{dh_{\pm lr}}{d\xi}$$
$$+\left[\frac{2\sigma_{+}^{2}h_{+lr}^{3}}{k_{\pm}^{2}}-\frac{2\sigma_{+}qh_{\pm lr}\epsilon_{\pm lR}}{k_{\pm}^{2}}-\frac{q^{2}\epsilon_{\pm lI}^{2}}{2h_{\pm lr}k_{\pm}^{2}}-\frac{q}{k_{\pm}}\frac{d\epsilon_{\pm lI}}{d\xi}\right]=0, \tag{3.22}$$

$$\frac{d^{2}h_{\pm 3r}}{d\xi^{2}}+\left(\frac{1}{k_{\pm}}\frac{dk_{\pm}}{d\xi}+\frac{2\sigma_{\pm}\sum h_{\pm li}}{k_{\pm}}-\frac{\sum dh_{\pm lr}/d\xi}{\sum h_{\pm lr}}\right)\frac{dh_{\pm 3r}}{d\xi}$$
$$+\left[\frac{2\sigma_{\pm}^{2}}{k_{\pm}^{2}}(\sum h_{\pm lr}^{2}+h_{\pm 1r}h_{\pm 2r}+h_{\pm 1i}h_{\pm 2i})h_{\pm 3r}\right.$$
$$-\frac{\sigma_{\pm}q}{k_{\pm}^{2}}(\sum\epsilon_{\pm lR}h_{\pm 3r}+\sum h_{\pm lr}\epsilon_{\pm 3R}-\sum h_{\pm li}\epsilon_{\pm 3I})$$
$$+\frac{q}{k_{\pm}}\left(\frac{\sum dh_{\pm lr}/d\xi}{\sum h_{\pm lr}}\epsilon_{\pm 3I}-\frac{d\epsilon_{\pm 3I}}{d\xi}\right)$$
$$\left.-\frac{\sigma_{\pm}}{k_{\pm}}\frac{\sum dh_{\pm lr}/d\xi}{\sum h_{\pm lr}}\sum h_{\pm li}h_{\pm 3r}\right]=0. \tag{3.23}$$

Using equations (3.10) and (3.20), we obtain

$$g_{\pm r}=\ln\left[(\sqrt{(h_{\pm 1r}h_{\pm 2r})}/k_{\pm})/(\sqrt{(h_{\pm 1r}h_{\pm 2r})}/k_{\pm})_{(\chi=\eta=0)}\right]$$
$$-\int_{0}^{\xi}\sum(\epsilon_{\pm aI}+\epsilon_{lI}/2h_{\pm lr})k_{\pm}^{-1}\,d\xi. \tag{3.24}$$

Consequently, we obtain the following expressions for the axial intensities.

$$I_{\pm a}=I_{\pm 0}[(\sqrt{(h_{\pm 1r}h_{\pm 2r})}/k_{\pm})/(\sqrt{(h_{\pm 1r}h_{\pm 2r})}/k_{\pm})_{(\chi=\eta=0)}]$$
$$\times \exp\left[-\int_0^{\xi}\sum(\epsilon_{\pm aI}+\epsilon_{\pm lI}/2h_{\pm lr})k_{\pm}^{-1}\,d\xi\right], \quad (3.25)$$

where the initial axial intensities $I_{\pm 0}$ are

$$I_{\pm 0}=(\beta\Gamma_{\pm})_{(\chi=\eta=\xi=0)}E_{\pm 0}^2. \quad (3.26)$$

Thus, using the expression (3.25) in evaluating $\epsilon_{\pm L}$ and $d\epsilon_{\pm L}/d\xi$, we can solve equations (3.22) and (3.23) for $h_{\pm Lr}$. In general, they cannot be solved analytically and have to be solved numerically; the Runge–Kutta method [22] is most suitable for the purpose. In the existing literature [11–16], $h_{\pm 1r}^{-1/2}=f_{\pm x}$ and $h_{\pm 2r}^{-1/2}\equiv f_{\pm y}$ are known as the beamwidth parameters (if $h_{\pm lr0}=1$). However, it will be more convenient here to call $h_{\pm Lr}$ as the beamwidth parameters. The boundary conditions, corresponding to plane wavefronts for both the modes, are

$$h_{\pm lr}(\xi=0)=h_{\pm lr0}, \quad (3.27)$$

$$h_{\pm 3r}(\xi=0)=0, \quad (3.28)$$

$$(dh_{\pm Lr}/d\xi)_{(\xi=0)}=0. \quad (3.29)$$

Effects of plasma inhomogeneity [23] and laser absorption [20] on laser self-focusing have already been investigated in detail in our previous papers. In this paper, we shall not consider these effects; hence, we omit the terms containing $dk_{\pm}/d\xi$, $\epsilon_{\pm aI}$ and $\epsilon_{\pm LI}$ in equations (3.22), (3.23) and (3.25). (Note that $dk_{\pm}/d\xi$ does not vanish even in an axially homogeneous plasma if the laser beam modifies the plasma-concentration; however, the electromagnetic field induced $dk_{\pm}/d\xi$ is generally negligible when $w_{\pm}\ll 1$). In equation (3.23), the most important term besides $d^2h_{\pm 3r}/d\xi^2$ is $\sigma_{\pm}q\sum h_{\pm lr}\epsilon_{\pm 3R}/k_{\pm}^2$; the other terms being quite small shall be neglected in order to simplify the analysis. Consequently, we obtain

$$\frac{d^2h_{\pm lr}}{d\xi^2}-\frac{3}{2h_{\pm lr}}\left(\frac{dh_{\pm lr}}{d\xi}\right)^2+\left[\frac{2\sigma_{\pm}^2h_{\pm lr}^3}{k_{\pm}^2}-\frac{2\sigma_{\pm}qh_{\pm lr}\epsilon_{\pm lR}}{k_{\pm}^2}\right]=0, \quad (3.30)$$

$$\frac{d^2h_{\pm 3r}}{d\xi^2}-\frac{\sigma_{\pm}q}{k_{\pm}^2}(h_{\pm 1r}+h_{\pm 2r})\epsilon_{\pm 3R}=0, \quad (3.31)$$

$$I_{\pm a}=I_{\pm 0}(h_{\pm 1r}h_{\pm 2r}/h_{\pm 1r0}h_{\pm 2r0})^{1/2}. \quad (3.32)$$

In the absence of a static magnetic field [20, 24], the right- and left-handedly polarized modes are not separate and we have

$$\frac{d^2h_{lr}}{d\xi^2}-\frac{3}{2h_{lr}}\left(\frac{dh_{lr}}{d\xi}\right)^2+\frac{2h_{lr}}{k^2}(h_{lr}^2-q\epsilon_{lr})=0, \quad (3.33)$$

$$h_{3r}=0. \quad (3.34)$$

According to equation (3.33), when $w=w(\chi=\eta=\xi=0)$ and $h_{lr0}=1$, a laser beam diverges monotonically if $I_0<1/qw$, gets self-trapped if $I_0=I_{\text{st low}}$ or $I_{\text{st upp}}$ where $I_{\text{st low, upp}}$ are solutions of the equation $(-R\,dP_a/dI_a)_{(I_a=I_{st})}=1/qw$, starts diverging but then oscillates periodically if $1/qw<I_0<I_{\text{st low}}$ or

$I_0 > I_{st\ upp}$, and starts converging but then oscillates periodically if $I_{st\ low} < I_0 < I_{st\ upp}$.

In the presence of a static magnetic field, $h_{\pm Lr}$ are all different and $h_{\pm 3r} \neq 0$ on account of the direct coupling between the two modes. Consequently, the axial symmetry of the initial intensity distributions of the two modes is lost as the two modes propagate inside the plasma. Still, however, it is possible to have $h_{\pm lr} = h_{\pm lr0}$ and $h_{\pm 3r} = 0$ (i.e. self-trapping of both the modes) provided

$$\sigma_{\pm 0} h_{\pm lr0}^2 = q\epsilon_{\pm lr0}. \tag{3.35}$$

This modified condition (for self-trapping) is more stringent than the condition $I_0 = I_{st}$ mentioned above in the case of absence of a static magnetic field. Indirect coupling arising on account of the intensity dependence of $\epsilon_\pm$ leads to cross-focusing [16] and aperiodic oscillations of the two modes. Faraday rotation and ellipticity [16] can be easily evaluated once we have evaluated $h_{\pm Lr}$, and hence we shall not discuss them further in the present investigation.

4. Results and discussion

Figure 1 shows the variations of

$$\psi_{\pm l} \equiv \sigma_\pm h_{\pm lr0}^2 / q\epsilon_{\pm lr0} (I_a = I_{st}) \tag{4.1}$$

versus

$$I_0 \equiv I_{+0} + I_{-0} \tag{4.2}$$

corresponding to the relativistic mass variation mechanism [17], ponderomotive

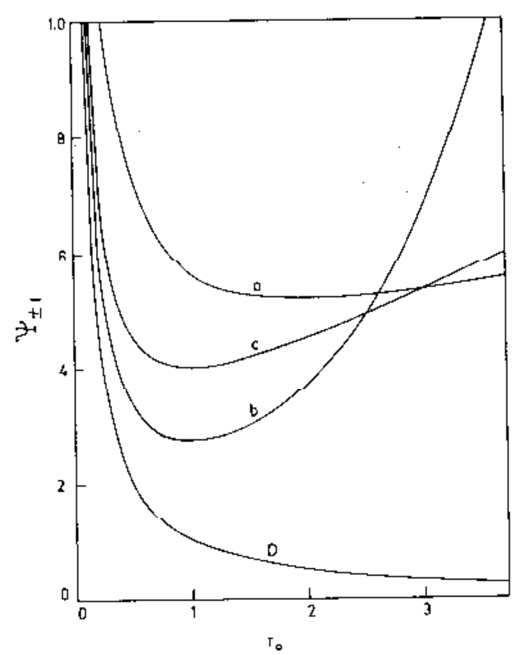

Figure 1. Variations of $\psi_{\pm l}$ with I_0 corresponding to (*a*) relativistic mass variation mechanism, (*b*) ponderomotive force mechanism, (*c*) collisional heating mechanism and (*d*) absence of non-linearity.

force mechanism [18], collisional heating mechanism [19] and absence of nonlinearity [20] (i.e. $\epsilon_{\pm lr0}=w$) for the curves labelled a, b, c and D respectively. If the values of I_0 and $(\sigma_{\pm} h_{\pm lr0}{}^2/qw)$ lie below the curve D, then the $\pm$ modes diverge monotonically so that $h_{\pm lr} < h_{\pm lr0}$. If they lie on the curve a (or b or c as the case may be), then the $\pm$ modes get self-trapped so that $h_{\pm lr} = h_{\pm lr0}$. If they lie in between the curves a (or b or c) and D, then the $\pm$ modes start diverging but then oscillate (not necessarily periodically) so that $dh_{\pm lr}/d\xi < 0$ at $\xi = 0$. If they lie above the curve a (or b or c), then the $\pm$ modes start converging but then oscillate so that $dh_{\pm lr}/d\xi > 0$ at $\xi = 0$. Aperiodicity in the oscillations of $h_{\pm lr}$ arises because of the indirect coupling between the two modes as represented by equation (4.2).

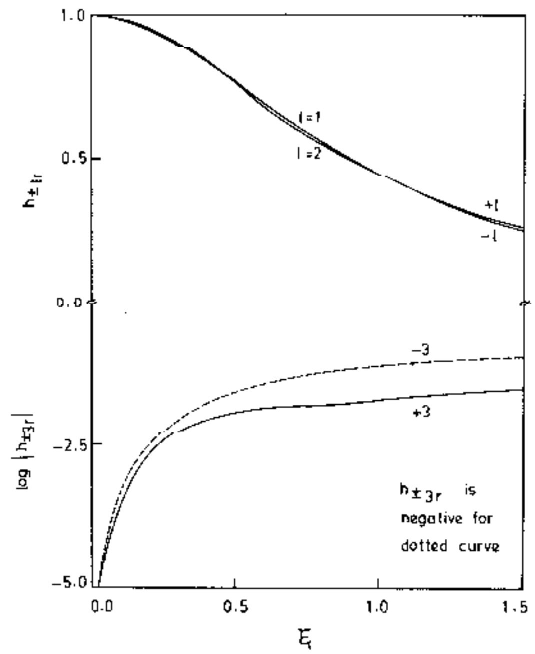

Figure 2. Variations of $h_{\pm lr}$ with ξ corresponding to $\gamma=0{\cdot}4$, $q=400$, $w=0{\cdot}2$, $h_{\pm lr0}=1$, and $I_{\pm 0}=10^{-5}$. (In figures 2–8, the ponderomotive force mechanism has been considered.)

We have solved equations (3.30) and (3.31) by the Runge–Kutta method [22], for the ponderomotive force mechanism and have presented the results in figures 2–8. Results corresponding to the relativistic mass variation and collisional heating mechanisms are qualitatively not much different and have, therefore, not been presented here.

Figures 2–4 correspond to $\gamma=0{\cdot}4$, $q=400$, $w=0{\cdot}2$, $h_{\pm lr0}=1$, and $I_0=10^{-5}$ for figure 2, 0·1 for figure 3, 10 for figure 4. Note that generally, but not necessarily, $h_{+3r} \simeq -h_{-3r}$. Though the values of $h_{\pm 3r}$ are very small as compared to $h_{\pm lr}$, they can be of interest in certain cases. Corresponding to nonvanishing $h_{\pm 3r}$, we have different values for $h_{\pm lr}$. The differences between $h_{\pm lr}$ are most significant when the beam converges. Also note that

$$|h_{\pm 1r}-h_{+2r}| < |h_{-1r}-h_{-2r}| < |h_{-1r}-h_{-1r}|. \tag{4.3}$$

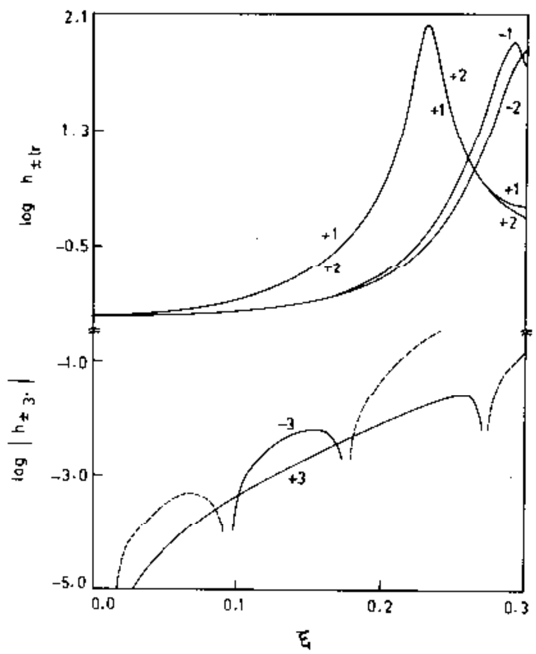

Figure 3. As figure 2, except that $I_{\pm 0} = 0{\cdot}1$.

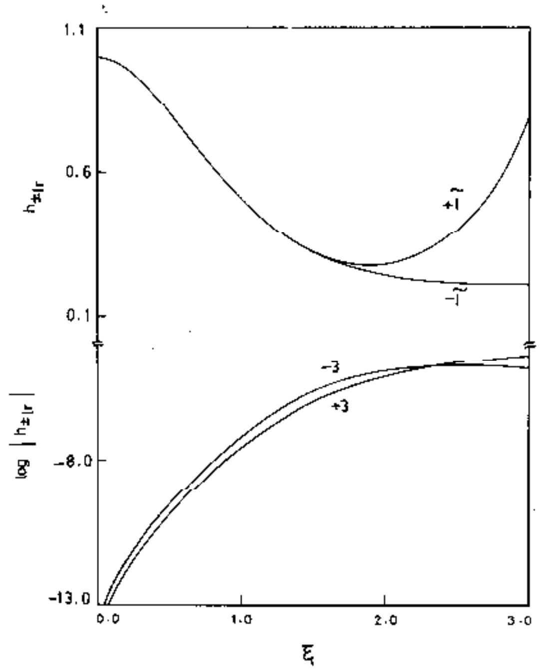

Figure 4. As figure 2, except that $I_{\pm 0} = 10$.

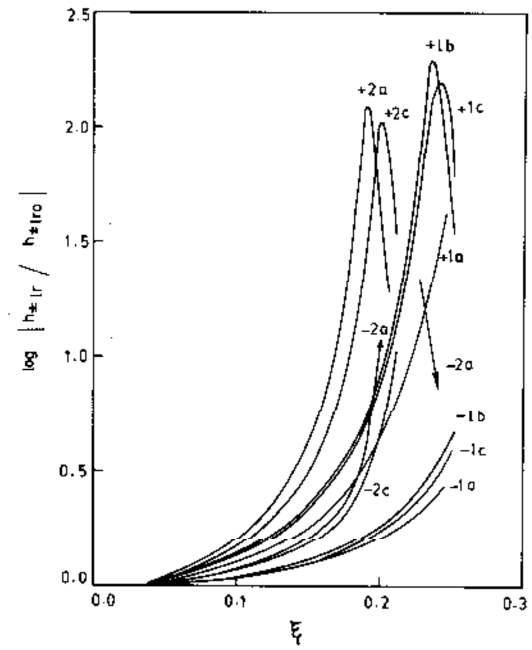

Figure 5. Variations of $h_{\pm lr}$ with ξ corresponding to $\gamma=0{\cdot}4$, $q=400$, $w=0{\cdot}2$, $h_{\pm 1r0}=0{\cdot}75$, $h_{+2r0}=(a)$ 1·5, (*b*) 0·75, (*c*) 1·5, $h_{-1r0}=(a)$ 0·75, (*b*) 1·5, (*c*) 1·5, $h_{-2r0}=(a)$ 1·5, (*b*) 1·5, (*c*) 0·75, and $I_{\pm 0}=0{\cdot}3/\sqrt{8}$. (In figures 5–8, the direct coupling between the ± modes has been neglected.)

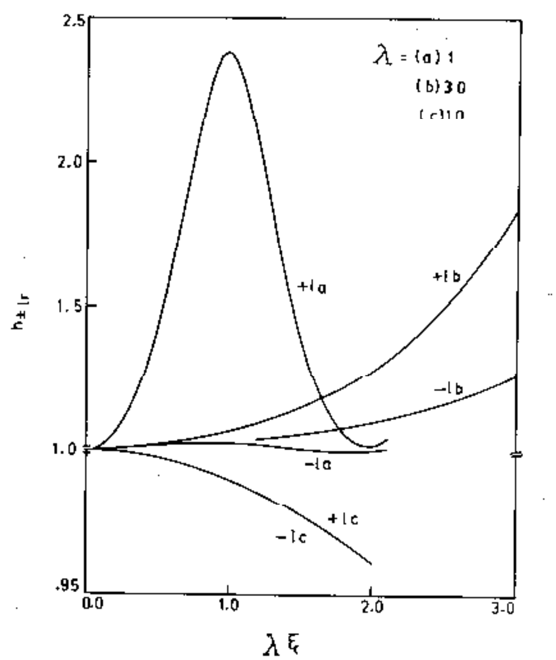

Figure 6. Variations of $h_{\pm lr}$ with ξ corresponding to $\gamma=0{\cdot}4$, $q=400$, $w=0{\cdot}2$, $h_{\pm lr0}=1$, and $I_{+0}=(a)$ 10^{-5}, (*b*) 0·1, (*c*) 10, $I_{-0}=(a)$ 10, (*b*) 1, (*c*) 0·1.

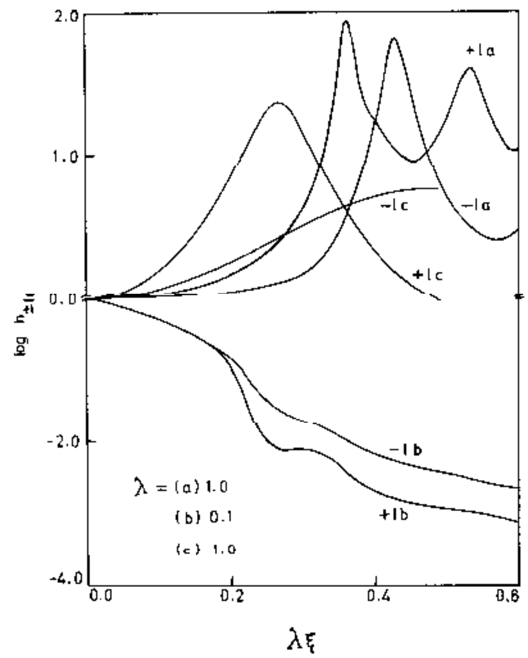

Figure 7. Variations of $h_{\pm 1r}$ with ξ corresponding to $\gamma = 0{\cdot}4 + 0{\cdot}5\chi^2 + 0{\cdot}5\eta^2$, $q = 400$, $T_0 = (a)\ T_{00}$, $(b)\ T_{00}$, $(c)\ T_{00}(1 + 0{\cdot}5\chi^2 + 0{\cdot}5\eta^2)$, $w = (a)\ 0{\cdot}2$, $(b)\ 0{\cdot}2$, $(c)\ 0{\cdot}2 + 0{\cdot}5\chi^2 + 0{\cdot}5\eta^2$, $h_{\pm lr0} = 1$, and $I_{\pm 0} = (a)\ 0{\cdot}1$, $(b)\ 10$, $(c)\ 1$.

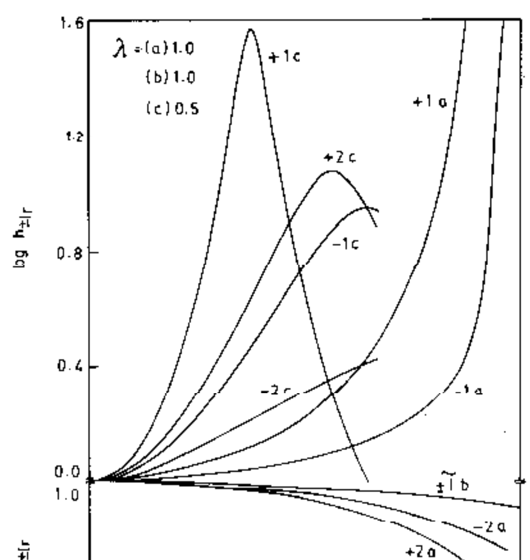

Figure 8. Variations of $h_{\pm lr}$ with ξ corresponding to $\gamma = 0{\cdot}4 + 0{\cdot}2\chi^2 + 0{\cdot}8\eta^2$, $q = 400$, $T_0 = (a)\ T_{00}$, $(b)\ T_{00}$, $(c)\ T_{00}(1 + 0{\cdot}3\chi^2 + 0{\cdot}7\eta^2)$, $w = (a)\ 0{\cdot}2$, $(b)\ 0{\cdot}2$, $(c)\ 0{\cdot}2 + 0{\cdot}7\chi^2 + 0{\cdot}3\eta^2$, $h_{\pm lr0} = 1$, and $I_{+0} = (a)\ 0{\cdot}1$, $(b)\ 10$, $(c)\ 1$.

In figures 5–8, we have neglected the direct coupling between the two modes so that $h_{\pm 3r} = 0$.

Figure 5 corresponds to $\gamma = 0{\cdot}4$, $q = 400$, $w = 0{\cdot}2$, $h_{+1r0} = 0{\cdot}75$, $h_{+2r0} = 1{\cdot}5$ for (a) and (c), 0·75 for (b), $h_{-1r0} = 0{\cdot}75$ for (a), 1·5 for (b) and (c), $h_{-2r0} = 1{\cdot}5$ for (a) and (b), 0·75 for (c), and $I_{\pm 0} = 0{\cdot}3/\sqrt{8}$. The curves show that the effect of any change in the initial beamwidth is quite appreciable. When one mode is focused, the other one can be defocused if the initial beamwidths for the two modes are different. Similarly, when a mode is focused along the X-axis, it can be defocused along the Y-axis if the initial beamwidths along the two axes are different. Such a difference in behaviour can influence the process of Faraday rotation [16] very appreciably.

Figure 6 corresponds to $\gamma = 0{\cdot}4$, $q = 400$, $w = 0{\cdot}2$, $h_{\pm lr0} = 1$, and $I_{+0} = 10^{-5}$ for (a), 0·1 for (b), 10 for (c), $I_{-0} = 10$ for (a), 1 for (b), 0·1 for (c). The extent of influence of indirect coupling on focusing/defocusing of the two modes is quite apparent from the graphs. Though $I_{+0} \ll I_{-0}$ for (a), the + mode is focused much more than the − mode. This is because of the fact that w_+ has $(1-\gamma)$ in the denominator, whereas w_- has $(1+\gamma)$.

Figures 7 and 8 correspond to $\gamma = 0{\cdot}4 + 0{\cdot}5\chi^2 + 0{\cdot}5\eta^2$ for figure 7, $0{\cdot}4 + 0{\cdot}2\chi^2 + 0{\cdot}8\eta^2$ for figure 8, $q = 400$, $T_0 = T_{00}$ for (a) and (b), $T_{00}(1 + 0{\cdot}5\chi^2 + 0{\cdot}5\eta^2)$ for 7 (c), $T_{00}(1 + 0{\cdot}3\chi^2 + 0{\cdot}7\eta^2)$ for 8 (c), $w = 0{\cdot}2$ for (a) and (b), $0{\cdot}2 + 0{\cdot}5\chi^2 + 0{\cdot}5\eta^2$ for 7 (c), $0{\cdot}2 + 0{\cdot}7\chi^2 + 0{\cdot}3\eta^2$ for 8 (c), $h_{\pm lr0} = 1$, and $I_{\pm 0} = 0{\cdot}1$ for (a), 10 for (b), 1 for (c). It is seen that non-uniformities [9] in the electron concentration and temperature and in the static magnetic field can change the behaviour of the two modes not only quantitatively but also qualitatively. Aperiodicity in the oscillations of the beamwidth parameters is considerably enhanced on account of non-uniformities. The effect of radial non-uniformities is most apparent in the case of $I_{\pm 0} = 10$ which roughly corresponds to $I_0 > I_{\mathrm{st\ upp}}$. Such an effect of radial non-uniformities in the plasma parameters can be easily understood in terms of the external focusing which has been extensively investigated in the context of optical fibres [25].

In the foregoing analysis, we have presented the parameters in their normalized (dimensionless) form. This presentation has an advantage over the presentation in their absolute (dimensional) form. As can be seen from figure 1 or from the analysis in § 3, identical end results may be obtained for several combinations of values of the absolute parameters. However, all possible combinations of values of the absolute parameters giving identical end results may be incorporated in a single combination of values of the normalized parameters.

While correlating our theory with an experiment, it is necessary to convert the presented normalized parameters into the required absolute parameters. This conversion can be easily carried out whenever necessary. As an illustration, however, let us find out a typical set of values of the absolute parameters which corresponds to the following set of normalized parameters chosen for figure 8 (c): $\gamma = 0{\cdot}4 + 0{\cdot}2\chi^2 + 0{\cdot}8\eta^2$, $q = 400$, $T_0 = T_{00}(1 + 0{\cdot}3\chi^2 + 0{\cdot}7\eta^2)$, $w = 0{\cdot}2 + 0{\cdot}7\chi^2 + 0{\cdot}3\eta^2$, $h_{\pm lr0} = 1$, and $I_{\pm 0} = 1$. Let us take $\omega = 10^{14}$/s, and $T_{00} = 10^6$ K. We then obtain:

$$r_0 = \text{initial beamwidth along } X \text{ or } Y \text{ axis} = \sqrt{(q)}c/\omega = 6 \times 10^{-3} \text{ cm},$$

$$E_{+0}^2 = 4m_0 k_B T_{00} \omega^2 (1-\gamma_0)^2 I_{+0}/(1-\gamma_0/2)e^2 = 10^{10} \text{ erg/cm}^3,$$

$$E_{-0}^2 = 4m_0k_B T_{00}\omega^2(1+\gamma_0)^2\, I_{-0}/(1-\gamma_0/2)e^2 = 5{\cdot}5\times 10^{10}\ \text{erg/cm}^3,$$

$$B_0 = \gamma m_0 c\omega/e = 2{\cdot}25\times 10^6(1 + x^2/1{\cdot}8\times 10^{-4}\ \text{cm}^2 + y^2/0{\cdot}45\times 10^{-4}\ \text{cm}^2)\ \text{G},$$

$$N_0 = wm_0\omega^2/4\pi e^2 = 6{\cdot}2\times 10^{17}(1 + x^2/1{\cdot}03\times 10^{-5}\ \text{cm}^2 + y^2/2{\cdot}4\times 10^{-5}\ \text{cm}^2)\ \text{cm}^{-3},$$

$$T_0 = 10^6(1 + x^2/1{\cdot}2\times 10^{-4}\ \text{cm}^2 + y^2/0{\cdot}51\times 10^{-4}\ \text{cm}^2)\ \text{K}.$$

Note that these values for the absolute parameters are realizable in an experimental set-up. Figure 8 (c) implies that the minimum distance at which a beamwidth parameter becomes minimum is $z_{(+1)} \cong 0{\cdot}2(\omega r_0^2/c) = 2{\cdot}4\times 10^{-2}$ cm. Both r_0 and $z_{(+1)}$ are much larger than the wavelength $c/\omega = 3\times 10^{-4}$ cm. In essence, therefore, the WKB approximation employed in the present investigation is justified.

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Cet article présente une étude de l'autofocalisation d'un faisceau laser elliptiquement gaussien traversant le champ magnétique statique d'un plasma magnétique à absorption non linéaire. On admet que la concentration d'électrons, la température et le champ magnétique statique sont non uniformes. Le couplage direct ainsi que le couplage indirect entre les modes de polarisation gauche et droit ont été pris en compte. Les couples d'équations pour les paramètres de divergence ont été établis en utilisant la méthode de Akhmanov. On voit que la symétrie axiale des répartitions d'intensité des deux modes n'est pas conservée en présence du couplage direct. Les variations des paramètres de divergence devient considérablement apériodiques si l'on tient compte du couplage indirect et de la non uniformité dans les paramètres de plasma. Bien que l'étude soit spécifiquement relative à un plasma magnétique, l'analyse présentée ici peut être aisément étendue à tout autre milieu anisotrope.

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