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Self-focusing of a laser pulse in a transient plasma

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Following Akhmanov's approach, self-focusing of a laser pulse in a transient plasma has been studied. The beamwidth parameter and hence the laser intensity and the frequency shift (time derivative of the phase) have been evaluated as a function of time and the distance of propagation. It is seen that the time dependence of the axial intensity changes appreciably as the pulse propagates. The present investigation is restricted to a pulse whose incident intensity has Gaussian radial dependence.

1. Introduction

The phenomenon of self-focusing of laser beams in plasmas (Sodha *et al.* 1974, 1976*a*) and in other nonlinear media (Akhmanov *et al.* 1972; Shen 1975; Svelto 1974) has been a subject of intense investigations in recent times. The phenomenon is quite significant and is relevant to important problems like laser-driven fusion (Brueckner & Jorna 1974). It influences other nonlinear phenomena like soliton formation (Patel 1977) and parametric instabilities (Patel 1977, personal communication; Sodha *et al.* 1977) which frequently occur in plasmas. Consideration of the transient nature of the laser beam and the medium leads to interesting results. Thus Lugovoi & Prokhorov (1974) interpreted the filamentary propagation of a laser pulse as the longitudinal motion of the intensity maxima. Using a rather phenomenological and numerical approach, Feit & Fleck (1976) have studied the variation of the axial intensity of a laser pulse in a steady-state plasma with time and the distance of propagation along the axis. Following Akhmanov *et al.* (1972), Sodha *et al.* (1976*b*) have studied the variation of the intensity of a laser pulse in a transient plasma with time and the distance of propagation and thus interpreted some of the results of the experiments carried out by Eremin *et al.* (1972).

In the present investigation, following Akhmanov *et al.* (1972), we have investigated the distortion of a laser pulse in a transient plasma. In §2, the expression for the dielectric constant (Ginzburg 1970) has been presented. In §3 the wave equation for a plane polarized electric field has been solved in the WKB and paraxial approximations and using the concept of beamwidth parameter (Akhmanov *et al.* 1972; Sodha *et al.* 1976*a*). In §4, numerical results along with a discussion have been presented.

It is convenient to use the dimensionless intensity

$$I \equiv \beta \mathbf{E}^* . \mathbf{E}, \tag{1.1}$$

where β is a mechanism-dependent constant to be defined later. It is well known (Sodha & Tripathi 1977*a, b*) that when the incident axial intensity

$$I_0 \equiv I_{(r=0,\, z=0)} \tag{1.2}$$

equals the lower or upper self-trapping intensity, $I_{\mathrm{st\,l}}$ or $I_{\mathrm{st\,u}}$, a laser beam can be self-trapped in a steady-state (and homogeneous non-absorptive) plasma; for $I_{\mathrm{st\,l}} < I_0 < I_{\mathrm{st\,u}}$, the axial intensity

$$I_a \equiv I_{(r=0,\, z)} \tag{1.3}$$

varies periodically between I_0 and a maximum value; for $I_0 > I_{\mathrm{st\,u}}$, I_a varies periodically between I_0 and a minimum value; and for $I_0 < I_{\mathrm{st\,l}}$, I_a decreases with the distance of propagation. We have, however, shown here and in concurrent investigations (Sodha, Patel & Kaushik; Tewari & Patheja, personal communications) that for $I_L < I_0 < I_{\mathrm{st\,l}}$, I_a varies periodically between I_0 and a minimum value; I_L is the self-trapping intensity predicted by the self-focusing theory based on the approximation (Sodha *et al.* 1974)

$$\epsilon \simeq \epsilon_{(I=0)} + I[\partial\epsilon/\partial I]_{(I=0)}. \tag{1.4}$$

The present investigation is seen partially to support Lugovoi & Prokhorov's (1974) interpretation. The variations of the frequency shift and the axial intensity are uncorrelated, and the time dependence of the axial intensity changes appreciably as the laser pulse propagates. Thus a laser pulse with Gaussian time dependence of I_0 displays increasing non-Gaussian time dependence of I_a.

2. Dielectric constant

For a quasi-monochromatic electric field expressed by

$$\mathbf{E}(r,z,t) = \mathscr{E}(r,z,t)\exp(i\omega t), \tag{2.1}$$

the displacement vector in a plasma free from any non-local effects is given by (Sodha *et al.* 1974)

$$\mathbf{D}(r,z,t) = [\epsilon(\omega,r,z,t)\,\mathscr{E}(r,z,t) - i(\partial\epsilon(\omega,r,z,t)/\partial\omega)\,(\partial\mathscr{E}(r,z,t)/\partial t)]\exp(i\omega t). \tag{2.2}$$

When the electromagnetic field frequency ω is much greater than the collision frequency ν and the relaxation effects are negligible, the dielectric constant of a plasma is given by (Ginzburg 1970; Sodha *et al.* 1976*a*)

$$\epsilon = 1 - wP, \tag{2.3}$$

where
$$w \equiv 4\pi N_0 e^2/m\omega^2, \tag{2.4}$$

N_0 is the electron concentration in the absence of the field, m is the electron rest mass and P represents the field-induced variation in the electron concentration (or mass). The ponderomotive force mechanism (Kaw *et al.* 1973; Sodha *et al.* 1976*a*) leads to

$$P = \exp(-I) \tag{2.5}$$

with the mechanism-dependent normalization constant

$$\beta \equiv I/\mathbf{E}^*.\mathbf{E} = e^2/4mk_B T_0 \omega^2, \tag{2.6}$$

where T_0 is the electron temperature in the absence of the field.

We shall assume the intensity I to have Gaussian radial dependence:

$$I = I_a \exp(-r^2/r_0^2 f^2) \tag{2.7}$$

'at least' in the paraxial region defined by

$$r \ll r_0 f. \tag{2.8}$$

Here $f(z)$ is the beamwidth parameter to be defined later. Using (2.3), (2.5) and (2.7) in the paraxial region, we get

$$\epsilon \simeq \epsilon_a - \epsilon_2 r^2/r_0^2 f^2, \tag{2.9}$$

where

$$\epsilon_a = 1 - wP_a, \tag{2.10}$$

$$\epsilon_2 = wP_2, \tag{2.11}$$

$$P_2 = I_a P_a = I_a \exp(-I_a). \tag{2.12}$$

3. Pulse propagation

When $k^2\epsilon \gg |\nabla^2\epsilon|$, a plane-polarized electric field $\mathbf{E}$ in a cylindrically symmetric plasma is governed by the scalar wave equation (Sodha *et al.* 1974)

$$\frac{\partial^2 E}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial E}{\partial r} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = 0. \tag{3.1}$$

For a slightly diverging/converging beam, in the WKB approximation, we may write

$$E = A_0[\epsilon_{a(z=0)}/\epsilon_a]^{\frac{1}{4}} \exp\left(i\omega t - i\int_0^z k\,dz - iS\right), \tag{3.2}$$

where

$$k \equiv \bar{\epsilon}_a^{\frac{1}{2}} k_0 \equiv \bar{\epsilon}_a^{\frac{1}{2}} \omega/c, \tag{3.3}$$

$$\bar{\epsilon} \equiv \epsilon + 2i\omega^{-1}\partial(wP)/\partial t + \omega^{-2}\partial^2(wP)/\partial t^2. \tag{3.4}$$

We shall neglect the term $2i\omega^{-1}\partial(wP)/\partial t$ in (3.4), so that k is real. Earlier investigations (Feit & Fleck 1976; Sodha *et al.* 1976*b*) have ignored the term $\omega^{-2}\partial^2(wP)/\partial t^2$ as well as $2i\omega^{-1}\partial(wP)/\partial t$.

Substituting (2.2), (2.9) and (3.2) into (3.1) and then separating the real and imaginary parts, it is seen that the real amplitude A_0 and eikonal S satisfy the coupled equations

$$\frac{\partial A_0}{\partial \xi} + \frac{1}{\bar{\epsilon}_a^{\frac{1}{2}}\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial S}{\partial \rho} + \frac{1}{\bar{\epsilon}_a^{\frac{1}{2}}}\frac{\partial S}{\partial \rho}\frac{\partial A_0}{\partial \rho} = 0, \tag{3.5}$$

$$\frac{\partial S}{\partial \xi} - \frac{1}{2A_0\bar{\epsilon}_a^{\frac{1}{2}}\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial A_0}{\partial \rho} + \frac{1}{2\bar{\epsilon}_a^{\frac{1}{2}}}\left(\frac{\partial S}{\partial \rho}\right)^2 - \frac{q\bar{\epsilon}_2\rho^2}{2\bar{\epsilon}_a^{\frac{1}{2}} f^2} = 0. \tag{3.6}$$

We have here replaced the old variables r, z, t by the new dimensionless variables

$$\rho \equiv r/r_0, \tag{3.7}$$

$$\xi \equiv z/k_0 r_0^2, \tag{3.8}$$

$$\tau \equiv \omega t - \int_0^z K\,dz, \tag{3.9}$$

where
$$K \equiv k_0[\bar{\epsilon}/\bar{\epsilon}_a]^{\frac{1}{2}}\,\partial(\omega\bar{\epsilon}^{\frac{1}{2}})/\partial\omega \tag{3.10a}$$
$$\simeq k_0\,\partial(\omega[\epsilon_a(I_a = I_0)]^{\frac{1}{2}})/\partial\omega, \tag{3.10b}$$
and, moreover, we have used the notation
$$q \equiv k_0^2 r_0^2. \tag{3.11}$$
The use of the approximate expression (3.10*b*) instead of the actual expression (3.10*a*) makes τ independent of r and z and thereby simplifies the analysis.

The incident intensity will be assumed to be radially Gaussian with mean beamwidth radius r_0, i.e.
$$I(\rho, \xi = 0, \tau) = I_0(\tau)\exp(-\rho^2), \tag{3.12}$$
$$I_0(\tau) = \beta E_0^2(\tau), \tag{3.13}$$
$$A_0(\rho, \xi = 0, \tau) = E_0(\tau)\exp(-\rho^2/2). \tag{3.14}$$
Following Akhmanov *et al.* (1972) and Sodha *et al.* (1976*a*), we then obtain, in the paraxial region,
$$I(\rho,\xi,\tau) = I_a(\xi,\tau)\exp(-\rho^2/f^2(\xi,\tau)), \tag{3.15}$$
$$I_a(\xi,\tau) = \beta E_0^2(\tau)f^{-2}(\xi,\tau)\,[\bar{\epsilon}_a(\xi = 0,\tau)/\bar{\epsilon}_a(\xi,\tau)]^{\frac{1}{2}}, \tag{3.16}$$
$$A_0(\rho,\xi,\tau) = E_0(\tau)f^{-1}(\xi,\tau)\exp(-\rho^2/2f^2(\xi,\tau)), \tag{3.17}$$
$$S(\rho,\xi,\tau) = S_a(\xi,\tau) + 2^{-1}\rho^2\bar{\epsilon}_a^{\frac{1}{2}}(\xi,\tau)\,\partial[\ln f(\xi,\tau)]/\partial\xi. \tag{3.18}$$
Substituting (3.17) and (3.18) into (3.6) and then equating the coefficients of ρ^0 and $\rho^2\bar{\epsilon}_a^{\frac{1}{2}}/2f$ with zero, we obtain the following equations for the axial eikonal $S_a(\xi,\tau)$ and the beamwidth parameter $f(\xi,\tau)$.
$$\frac{\partial S_a}{\partial\xi} = -\frac{1}{\bar{\epsilon}_a^{\frac{1}{2}}f^2}, \tag{3.19}$$
$$\frac{\partial^2 f}{\partial\xi^2} + \frac{\partial\ln\bar{\epsilon}_a^{\frac{1}{2}}}{\partial\xi}\frac{\partial f}{\partial\xi} = \frac{1}{\bar{\epsilon}_a}\left[\frac{1}{f^3} - \frac{qwP_2}{f}\right]. \tag{3.20}$$
The boundary conditions are
$$S_a(\xi = 0,\tau) = 0, \tag{3.21}$$
$$f(\xi = 0,\tau) = 1, \tag{3.22}$$
$$(\partial f/\partial\xi)_{(\xi=0,\tau)} = 0. \tag{3.23}$$
We shall assume the plasma to be homogeneous in the absence of the field and $\left(\int_0^z K dz\right)$ to be negligibly small so that w may depend only on τ, i.e. $w = w(\tau)$. Integrating (3.19) and $(2\bar{\epsilon}_a\,\partial f/\partial\xi)$ times (3.20), we obtain
$$S_a = -\int_0^{\xi}\bar{\epsilon}_a^{-\frac{1}{2}}f^{-2}d\xi, \tag{3.24}$$
$$(\partial f/\partial\xi)^2 = \bar{\epsilon}_a^{-1}\left[1 - f^{-2} - qw\int_1^{f^2}P_2 f^{-2}df^2\right]. \tag{3.25}$$

Integrating (3.20) numerically by the Runge–Kutta method (Scarborough 1966) or integrating (3.25) analytically when possible, we can find the beamwidth parameter $f(\xi, \tau)$.

In order to study the behaviour of f qualitatively, let us neglect the dispersion effects. Then (3.20) implies (Sodha & Tripathi 1977*a*, *b*) that $f(\xi, \tau) = 1$, i.e. the portion of the laser pulse at time τ is self-trapped (in a homogeneous non-absorptive plasma) provided $I_0(\tau) = I_{st}(\tau)$ such that

$$qw = [P_2(I_a = I_{st})]^{-1} = \exp(I_{st})/I_{st}. \tag{3.26}$$

For $qw < (qw)_{cr} = e$, there is no I_{st} to satisfy (3.26); for $qw = e$, $I_{st} = (I_{st})_{cr} = 1$; and for $qw > e$, (3.26) has two roots $I_{st\,l}$ and $I_{st\,u}$ which decrease and increase respectively as qw increases. For $I_{st\,l} < I_0 < I_{st\,u}$, $qwP_2(I_a = I_0) > 1$ and f starts decreasing; whereas either for $I_0 < I_{st\,l}$ or for $I_0 > I_{st\,u}$, $qwP_2(I_a = I_0) < 1$ and f starts increasing. For $I_0 < I_L$, $(qwP_2f^2) < 1$ for all f and hence f keeps increasing with ξ; whereas for $I_0 > I_L$, qwP_2f^2 can cross the value unity as f changes and hence f oscillates periodically between 1 and $f_{ex}(=f_{max}$ or $f_{min})$ with a period of $2\xi_{ex}$. Neglecting the effect of the term $(\partial \ln \bar{\epsilon}_a^{\frac{1}{2}}/\partial\xi)\,(\partial f/\partial\xi)$ present in (3.20), we get

$$I_L = (qw)^{-1}, \tag{3.27}$$

and approximating I_a as (I_0/f^2), we get

$$qw(\exp(-I_0 f_{ex}^{-2}) - \exp(-I_0)) + f_{ex}^{-2} - 1 = 0. \tag{3.28$'$}$$

For values of I_0 such that $(\partial^2 f/\partial\xi^2) \simeq 0$ when $f = (1+f_{ex})/2$, we get

$$f_{ex} \simeq 2[I_0/\ln(qwI_0)]^{\frac{1}{2}} - 1. \tag{3.28}$$

The frequency shift (Sodha *et al.* 1974, 1976*a*) normalized by the frequency ω, is defined as

$$\Delta(\rho, \xi, \tau) = -\partial S(\rho, \xi, \tau)/\partial\tau. \tag{3.29}$$

It is convenient to express Δ as

$$\Delta = \left(\Delta_{aI_0}\frac{\partial I_0}{\partial\tau} + \Delta_{aw}\frac{\partial w}{\partial\tau}\right) - \left(\Delta_{2I_0}\frac{\partial I_0}{\partial\tau} + \Delta_{2w}\frac{\partial w}{\partial\tau}\right)\rho^2, \tag{3.30}$$

where, using (3.18) and (3.29), we have

$$\Delta_{aI_0} = -dS_a/dI_0, \tag{3.31}$$

$$\Delta_{aw} = -dS_a/dw, \tag{3.32}$$

$$\Delta_{2I_0} = d[2^{-1}\bar{\epsilon}_a^{\frac{1}{2}}\,\partial(\ln f)/\partial\xi]/dI_0, \tag{3.33}$$

$$\Delta_{2w} = d[2^{-1}\bar{\epsilon}_a^{\frac{1}{2}}\,\partial(\ln f)/\partial\xi]/dw. \tag{3.34}$$

The knowledge of the dependence of the Δ coefficients Δ_{aI_0}, Δ_{aw}, Δ_{2I_0} and Δ_{2w} on (ρ, ξ, I_0, w) and the dependence of I_0 and w on τ, simplifies the study of the dependence of the Δ coefficients on (ρ, ξ, τ). Since the dispersion effects are generally negligible, the Δ coefficients do not depend explicitly on τ.

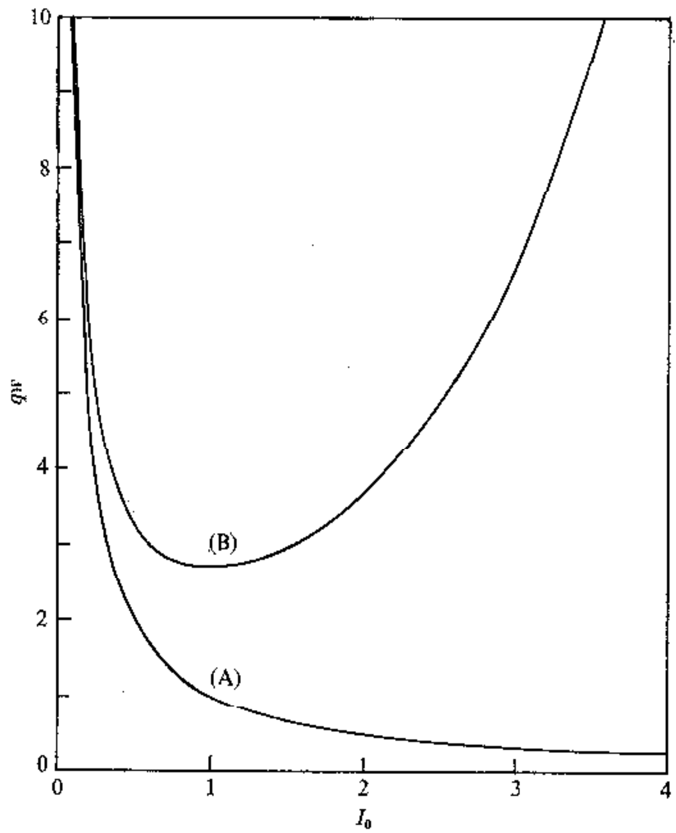

FIGURE 1. qw versus I_0 for I_0 = (A) I_L, (B) I_{st}.

4. Discussion

Figure 1 illustrates how the dimensionless parameter qw ($\equiv 4\pi N_0 e^2 r_0^2/mc^2$) and the dimensionless incident axial intensity I_0 should be chosen so that a laser beam can be self-trapped (in a homogeneous non-absorptive plasma): (A) according to the self-focusing theory based on the approximation (1.4) and (B) according to the self-focusing theory based on the ponderomotive force mechanism with the correct I dependence of ϵ. In other words, the curve A is for $qw = 1/I_L$ versus I_L whereas the curve B is for $qw = \exp(I_{st})/I_{st}$ versus I_{st}. For values of qw and I_0 lying below or on the curve A, the beam diverges monotonically (Sodha *et al.* 1974), i.e. $f \geqslant 1$. For values of qw and I_0 lying in between the curves A and B, the axial intensity I_a oscillates (Sodha, Patel & Kaushik, personal communication) between the incident axial intensity I_0 and a minimum value $I_{a(\min)}$, i.e.

$$f_{\max} \geqslant f \geqslant 1.$$

For values of qw and I_0 lying on the curve B, the beam becomes self-trapped (Sodha & Tripathi 1977*a*, *b*), i.e. $f = 1$. For values of qw and I_0 lying above the curve B, the axial intensity I_a oscillates (Sodha & Tripathi 1977*a*, *b*; Sodha,

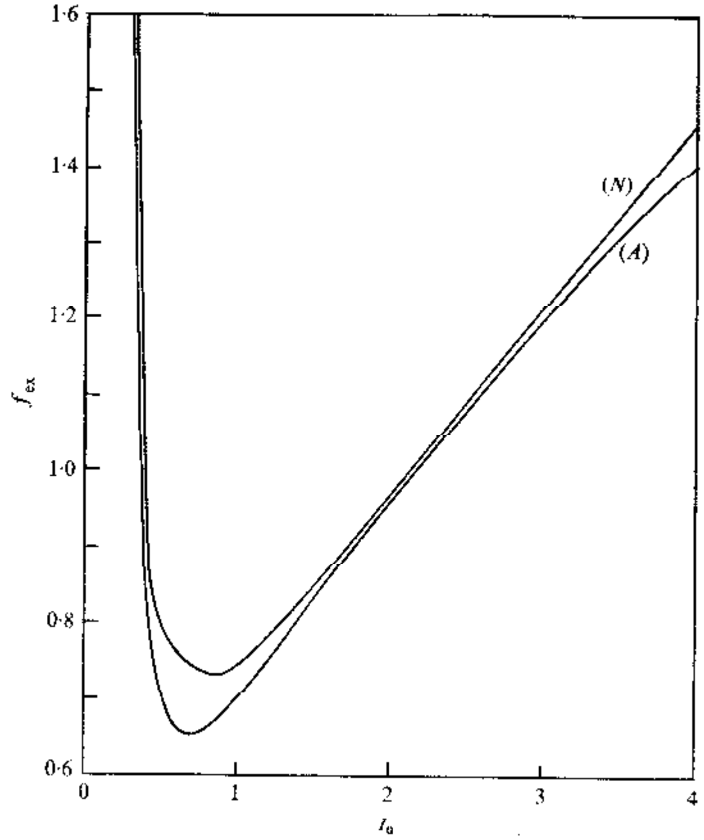

FIGURE 2. f_{ex} versus I_0 for $q = 100$ and $w = 0{\cdot}04$ as obtained (A) analytically, (N) numerically.

Patel & Kaushik; Tewari & Patheja, personal communications) between the incident axial intensity I_0 and a maximum value $I_{a(max)}$, i.e. $f_{min} \leqslant f \leqslant 1$.

Figures 2–8 correspond to $q = 100$ and $w = 0{\cdot}04$.

Figure 2 illustrates the variation of f_{ex} ($= f_{max}$ or f_{min}) with I_0 as obtained (A) analytically according to (3.28) and (N) from solving (3.20) numerically by the Runge–Kutta method (Scarborough 1966). This figure does indicate that $f_{max}(+\infty) \geqslant f \geqslant 1$ for $I_L < I_0 < I_{st\,l}$. Because of the approximations used in arriving at (3.28), the analytical results do not coincide with the actual numerical ones; however, the agreement between the two is tolerable.

Figure 3 represents the variation of ξ_{ex} (the dimensionless distance at which the axial intensity becomes minimum/maximum, i.e. f becomes f_{ex}) with I_0 as obtained numerically. The slope of the curve is large for $I_L < I_0 < I_{st\,l}$, but small for $I_0 > I_{st\,u}$. The non-zero slope of the curve for $I_{st\,l} < I_0 < I_{st\,u}$ implies, in the case of a laser pulse, the longitudinal motion of the foci where the instantaneous axial intensity becomes maximum. Lugovoi & Prokharov (1974) had interpreted the filamentary tracks observed along the propagation of a laser pulse in terms of such a longitudinal motion of foci. Such an interpretation would require

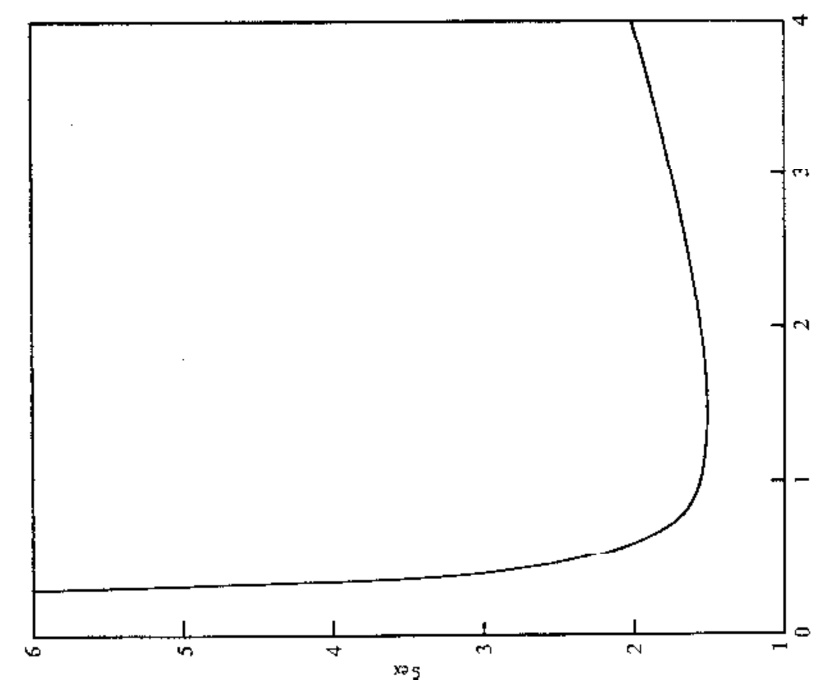

FIGURE 3. ξ_{ex} versus I_0 for $q = 100$ and $w = 0{\cdot}04$.

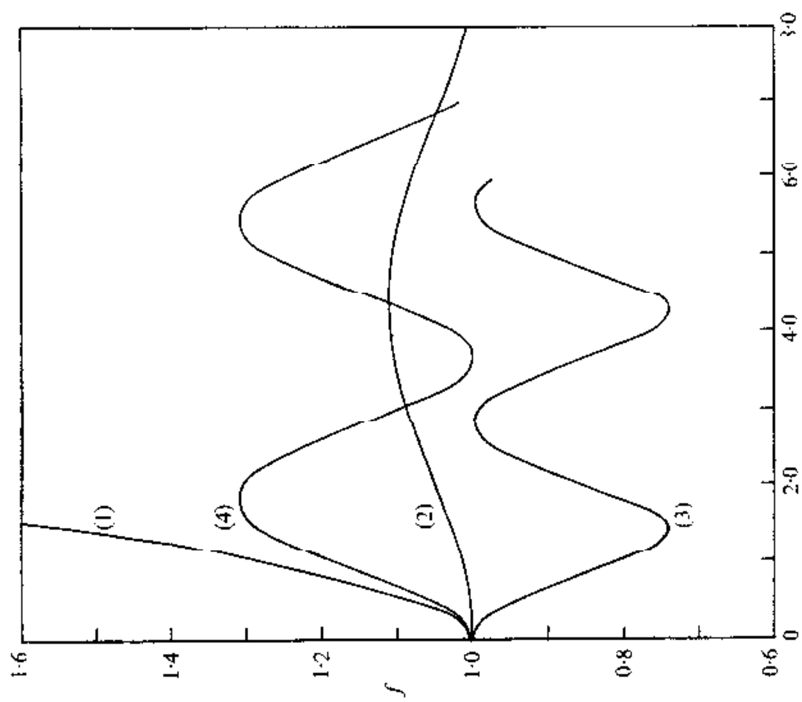

FIGURE 4. f versus ξ for $I_0 =$ (1) 0·1, (2) 0·34, (3) 1·0, (4) 3·4, $q = 100$ and $w = 0{\cdot}04$.

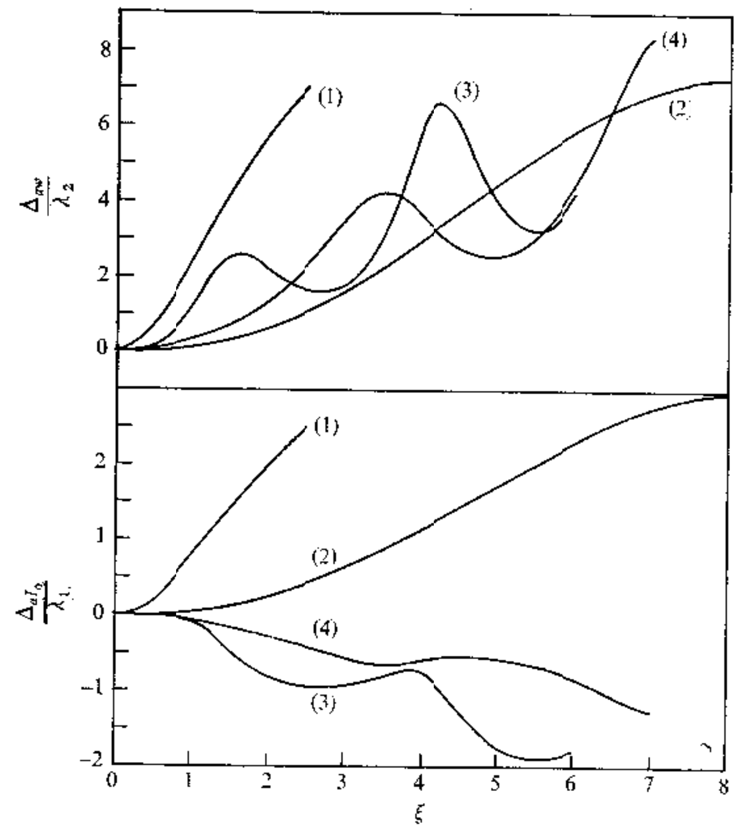

FIGURE 5. Δ_{aI_0}/λ_1 and Δ_{4w}/λ_2 versus ξ for $I_0 =$ (1) 0·1, (2) 0·34, (3) 1·0, (4) 3·4, $q = 100$ and $w = 0{\cdot}04$. $\lambda_1 =$ (1) 1, (2), 20, (3) 1, (4) 1 and $\lambda_2 =$ (1) 1, (2) 100, (3) 20, (4) 10.

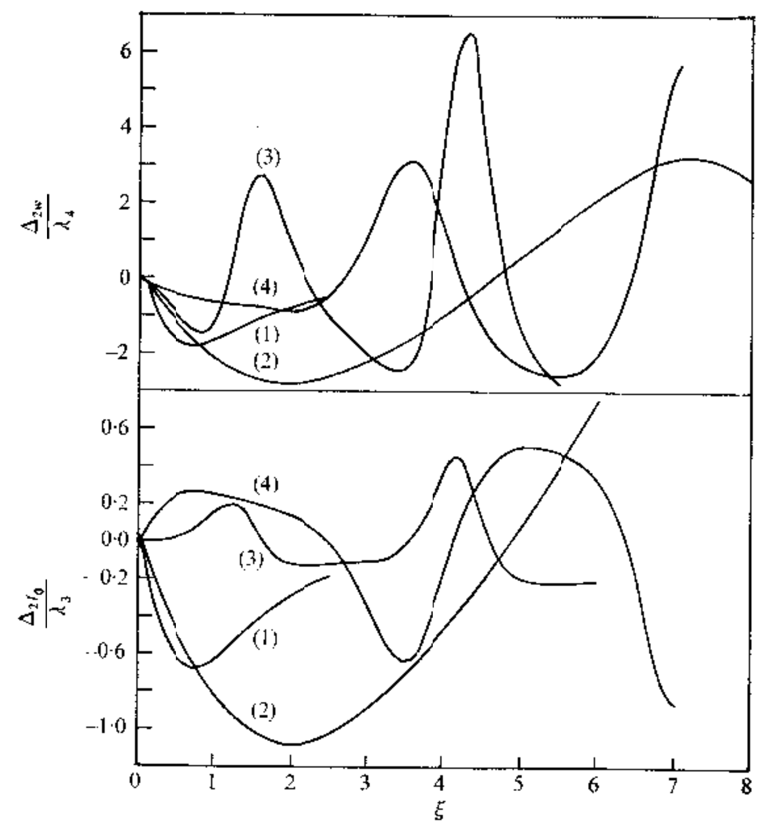

FIGURE 6. Δ_{2I_0}/λ_3 and Δ_{2w}/λ_4 versus ξ for $I_0 =$ (1) 0·1, (2) 0·34, (3) 1·0, (4) 3·4, $q = 100$ and $w = 0{\cdot}04$. $\lambda_3 =$ (1) 1, (2) 1, (3) 1, (4) $\frac{1}{4}$ and $\lambda_4 =$ (1) 1, (2) 5, (3) 10, (4) 5.

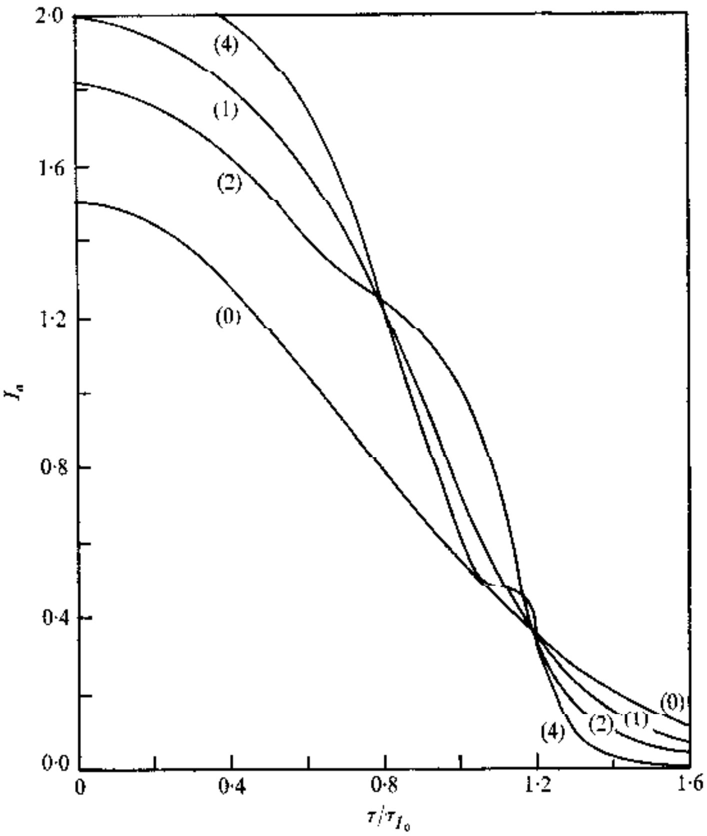

FIGURE 7. I_a versus τ/τ_{I_0} for $I_0 = 1{\cdot}5 \exp(-\tau^2/\tau_{I_0}^2)$, $q = 100$, $w = 0{\cdot}04$ and at $\xi =$ (0) 0, (1) 1, (2) 2, (4) 4.

$(d \ln f_{\min}/dI_0) \ll (d \ln \xi_{ex}/dI_0)$ for $I_{st\,l} < I_0 < I_{st\,u}$, whereas figures 2 and 3 show that $(d \ln f_{\min}/dI_0)$ is generally much greater than $(d \ln \xi_{ex}/dI_0)$ for $I_{st\,l} < I_0 < I_{st\,u}$. Hence the present investigation supports Lugovoi & Prokhorov's (1974) interpretation only partially.

Figure 4 illustrates the variation of f with ξ for $I_0 =$ (1) 0·1, (2) 0·34, (3) 1·0, (4) 3·4 which correspond to (1) $I_0 < I_L$, (2) $I_L < I_0 < I_{st\,l}$, (3) $I_{st\,l} < I_0 < I_{st\,u}$, (4) $I_0 > I_{st\,u}$. The figure confirms the foregoing qualitative discussion on the behaviour of f. The effect of temporal dispersion (Sodha *et al.* 1974) characterized by the term $[\omega^{-2}\partial^2(wP)/\partial t^2]$ in (3.4) is quite negligible and hence, in figure 4, the curve for a given value of I_0 does not explicitly depend upon τ.

Figure 5 represents the variation of the axial Δ coefficients Δ_{aI_0} and Δ_{aw} with ξ for $I_0 =$ (1) 0·1, (2) 0·34, (3) 1·0, (4) 3·4; figure 6 corresponds to the off axial Δ coefficients Δ_{2I_0} and Δ_{2w}. On account of the negligible effect of temporal dispersion, the curves do not explicitly depend upon τ. As expected, the curves do not show any apparent correlation with the variation of f represented in figure 4. The curves of the Δ coefficients are very useful in the calculation of the instantaneous frequency shift (Sodha *et al.* 1974, 1976*a*) for any type of time dependence of I_0 and w. Thus, for $q = 100$, $w = 0{\cdot}04$, $\rho = 0$, $\xi = 3$, $I_0 = 1{\cdot}0$, $(\partial I_0/\partial\tau) = 10^{-10}$

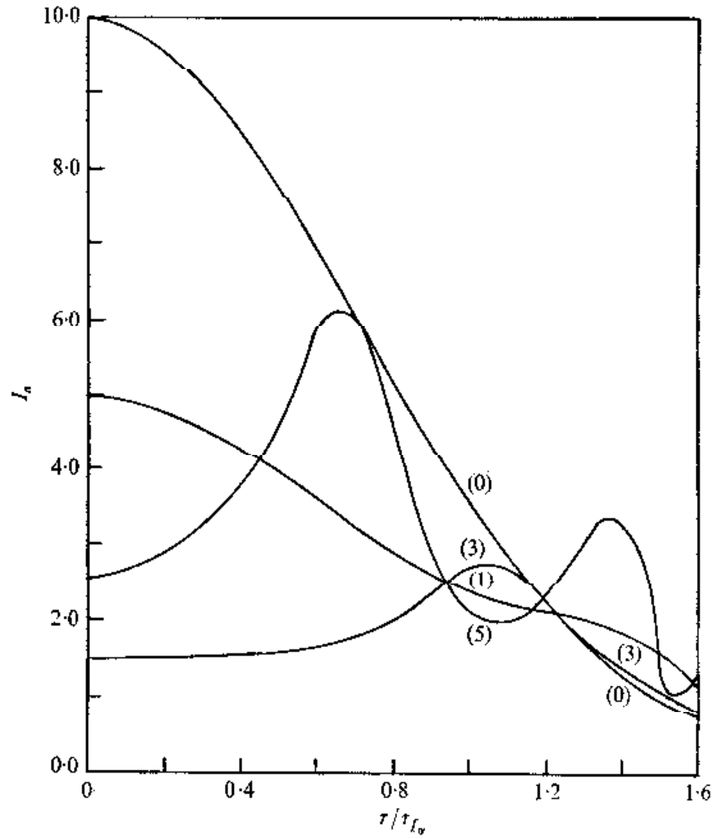

FIGURE 8. I_a versus τ/τ_{I_0} for $I_0 = 10\exp(-\tau^2/\tau_{I_0}^2)$, $q = 100$, $w = 0{\cdot}04$ and at $\xi = (0)\ 0,\ (1)\ 1,\ (3)\ 3,\ (5)\ 5$.

and $(\partial w/\partial\tau) = 10^{-14}$, we predict from the third graphs in figure 5 that the normalized frequency shift is

$$\Delta \simeq (-0{\cdot}93\times10^{-10}+36\times10^{-14}) = -0{\cdot}9264\times10^{-10}.$$

Thus, the frequency shift is in general negligibly small.

Figure 7 represents the variation of I_a with (τ/τ_{I_0}) for $\xi = (0)\ 0,\ (1)\ 1,\ (2)\ 2,\ (4)\ 4$ and $I_0 = 1{\cdot}5\exp(-\tau^2/\tau_{I_0}^2)$; $I_0(\tau=0) = 1{\cdot}5$ is less than the upper self-trapping intensity $I_{\text{st u}}$. Figure 8 represents the variation of I_a with (τ/τ_{I_0}) for $\xi = (0)\ 0,\ (1)\ 1,\ (3)\ 3,\ (5)\ 5$ and $I_0 = 10\exp(-\tau^2/\tau_{I_0}^2)$; $I_0(\tau=0) = 10$ is greater than $I_{\text{st u}}$. We note that the initial Gaussian time dependence of the intensity is distorted to such an extent that in many cases the pulse shape develops peaks. In earlier investigations (Sodha *et al.* 1976*a*, *b*; Eremin *et al.* 1972) restricted to small values of ξ and $I_0(\tau=0)$, the development of peaks was not observed. The sort of pulse-shape distortion observed in the present investigation can be easily interpreted on the basis of the double-valuedness of the self-trapping intensity. For high enough values of ξ, the curves in figure 8 show dips at $\tau = 0$ because $I_0(\tau=0)$ has been chosen to be greater than $I_{\text{st u}}$ in this case. This prediction may be easily checked from experiments with very intense laser pulses through plasmas.

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