Average field of a laser beam self-focused in a turbulent plasma

M. S. Sodha and L. A. Patel
Centre of Energy Studies, Indian Institute of Technology, New Delhi-110029, India

(Received 8 June 1979; accepted for publication 21 January 1980)

This paper presents an investigation on the spatial variation of the average field of a laser beam which is self-focused in a plasma having random fluctuations in its electron concentration. The analysis is valid for the arbitrary mechanism of self-focusing nonlinearity, for nonlinearly absorbing and nonuniform plasmas, and for elliptically Gaussian laser beams. The Novikov-Furutsu formalism has been followed to derive a parabolic equation for the average field, and the Akhmanov approach has been employed to solve this equation. It is concluded that the plasma turbulence tends to reduce the field intensity as the laser beam penetrates further. The self-focusing-induced oscillations of the beamwidth parameters and axial intensity become considerably aperiodic in the presence of turbulence; after a sufficient distance of propagation, an oscillating beamwidth parameter (axial intensity) starts increasing (decreasing) monotonically.

PACS numbers: 42.60.Kg,42.65.Jx,52.40.Db,84.40.Ed

I. INTRODUCTION

The subject of laser-beam propagation in plasmas is of great practical importance from the point of view of atmospheric communication[1] and thermonuclear fusion.[2] It is well-known that a laser beam can modify the parameters of the medium in such a way that it would self-focus[3-5] in the course of its propagation. It is also well-known that the plasma medium can exhibit turbulence which is characterized by random fluctuations in the electron concentration.[6] The simultaneous consideration of self-focusing and turbulence is, therefore, essential in dealing with the laser-beam propagation in plasmas. The atmospheric plasma researchers[1,6] have not considered the effect of self-focusing on the laser-beam propagation through turbulence. Recently some fusion-plasma researchers[7] have investigated the effect of turbulence on the laser self-focusing; however, since they have treated the turbulence as a perturbation, their investigations are valid only for a very limited range of turbulence.

In this paper, the spatial variation of the average field of a laser beam propagating in a plasma has been investigated by taking both the self-focusing and turbulence into account. In Sec. II, the expression for the dielectric constant of a turbulent plasma and its series expansion, valid in the paraxial region, have been presented. Though the ponderomotive-force mechanism has been considered for the purpose of illustration, the analysis is valid for the arbitrary mechanism of self-focusing nonlinearity. The plasma is allowed to be nonlinearly absorbing[8] and radially[8] or axially[9] nonuniform, and the transverse intensity profile of the laser beam is allowed to be elliptically Gaussian.[10] In Sec. III, an equation for the average field of the laser beam has been derived by following the Novikov-Furutsu formalism; it has been assumed that self-focusing nonlinearity does not distort the "qualitative form" of the turbulence, so that the Novikov-Furutsu formalism remains valid even in the presence of nonlinearity. By defining the effective components of the dielectric constant, the above-mentioned equation has been reduced to a form similar to the one encountered in NA. (The abbreviation NA denotes reference[8] to the paper "Effect of nonlinear absorption on self focusing of a laser beam in a plasma". Apart from some modifications related to the plasma turbulence and elliptical cross section, the notations used in the present investigation are the same as those in NA.) In Sec. IV, the average field has been expressed in terms of the beamwidth parameters[4,10] (along the x and y axes), which are governed by second-order ordinary coupled differential equations. In Sec. V, certain typical sets of parameters (in their normalized form) have been chosen and the corresponding numerical results have been presented.

It is evident from the present investigation that the plasma turbulence tends to reduce the field intensity as the laser beam penetrates further; unlike in a linear case,[1,6] it is not possible in the present case to define an absorption coefficient related to the turbulence. The self-focusing-induced oscillations of the beamwidth parameters and axial intensity become considerably aperiodic in the presence of turbulence; after a sufficient distance of propagation, an oscillating beamwidth parameter (axial intensity) starts increasing (decreasing) monotonically. The first focal length (i.e., the minimum distance at which the beamwidth parameter attains a local minimum value) increases as the level of turbulence is increased. Aperiodic oscillations followed by monotonic rise (fall) of the beamwidth parameter (axial intensity) are caused even by nonlinear absorption or axial inhomogeneity of the plasma. For an elliptically Gaussian laser beam, the beamwidth parameters along the x and y axes vary differently, and thereby the ellipticity of the laser beam cross section varies with the distance of propagation.

II. DIELECTRIC CONSTANT

The dielectric constant of a turbulent plasma at a frequency ω much greater than the collision frequency ν is given by

$$\epsilon = \epsilon_s - \epsilon_r, \tag{2.1}$$

$$\epsilon_s = 1 - WP - iVQ, \tag{2.2}$$

$$\epsilon_r = W\mathscr{P} + iV\mathscr{Q}. \tag{2.3}$$

 0021-8979/80/052381-06$01.10 © 1980 American Institute of Physics

As in NA,

$$W = 4\pi \langle N_0 \rangle e^2/m\omega^2, \tag{2.4}$$

$$V = 4\pi \langle N_0 \nu_0 \rangle e^2/m\omega^3, \tag{2.5}$$

$$P = \langle N \rangle / \langle N_0 \rangle, \tag{2.6}$$

$$Q = \langle N\nu \rangle / \langle N_0 \nu_0 \rangle, \tag{2.7}$$

$$\mathscr{P} = (N - \langle N \rangle)/\langle N_0 \rangle, \tag{2.8}$$

$$\mathscr{Q} = (N\nu - \langle N\nu \rangle)/\langle N_0 \nu_0 \rangle, \tag{2.9}$$

where e is the electron charge, m is the electron rest mass, N is the electron concentration, ν is the electron collision frequency; N_0 and ν_0 are N and ν, respectively, in the absence of the field; $\langle \; \rangle$ denotes the ensemble average of the quantity within the brackets. Note that $\mathscr{P}$ and $\mathscr{Q}$ denote the fluctuating components of N and $N\nu$, respectively.

The functions $P, Q, \mathscr{P}$, and $\mathscr{Q}$ depend on the normalized (dimensionless) field intensity

$$I = \beta \langle |\mathbf{E}|^2 \rangle. \tag{2.10}$$

Since the laser field is almost coherent (i.e., coherency > 0.5), the definition (2.10) may be written as

$$I = \beta |\langle \mathbf{E} \rangle|^2 \tag{2.11}$$

In order to simplify the analysis, it will be assumed that the field-induced redistribution of the plasma particles is not accompanied by any significant nonlocal or nonstationary processes. Then the fluctuating components of N and $N\nu$ vary almost proportionally with $\langle N \rangle$ and $\langle N\nu \rangle$, respectively. Hence

$$\mathscr{P} = \mathscr{P}_L P, \tag{2.12}$$

$$\mathscr{Q} = \mathscr{Q}_L Q, \tag{2.13}$$

where $\mathscr{P}_L$ and $\mathscr{Q}_L$ are $\mathscr{P}$ and $\mathscr{Q}$, respectively, in the absence of the field.

The aim of this analysis, in the present investigation, is to be valid for the arbitrary mechanism of self-focusing nonlinearity. As an illustration, however, a particular mechanism may be considered. Let the laser and plasma system be such that the redistribution of electrons and ions of the plasma (assumed to be fully ionized) is caused by the ponderomotive force. In this case,

$$\beta = e^2/4mK_B T_0 \omega^2, \tag{2.14}$$

$$P = \exp(-I), \tag{2.15}$$

$$Q = \exp(-bI), \tag{2.16}$$

where K_B is the Boltzmann constant, T_0 is the plasma temperature in the absence of the field, and $b = 2$ or 1 depending upon whether the electron collisions are with ions or neutrals.

The Akhmanov approach to be employed in Sec. IV requires the Maclaurin series expansion of the dielectric constant. Following NA, for an initially elliptically Gaussian laser beam, I is assumed to be given by

$$I = I_a \exp(-x^2/x_0^2 f_x^2 - y^2/y_0^2 f_y^2)$$
$$= I_a \exp(-h_{xr}\gamma_x^2 - h_{yr}\gamma_y^2). \tag{2.17}$$

The axial intensity I_a and the beamwidth parameters

$$f_\alpha \equiv h_{\alpha r}^{-1/2} \tag{2.18}$$

vary with the propagation distance z, but not with the transverse coordinates

$$\alpha \equiv \gamma_\alpha r_0, \tag{2.19}$$

where α stands for x and y. The paraxial region is characterized by the inequality

$$\alpha \ll \alpha_0 f_\alpha. \tag{2.20}$$

This inequality justifies the neglect of higher-order terms in the Maclaurin series expansion of the dielectric constant. The first-order expansion leads to

$$\epsilon_s = (\epsilon_{s0r} + i\epsilon_{s0i}) + \sum_\alpha (\epsilon_{s2r} + i\epsilon_{s2i}) h_{\alpha r} \gamma_\alpha^2, \tag{2.21}$$

$$\epsilon_L = (\epsilon_{L0r} + i\epsilon_{L0i}) + \sum_\alpha (\epsilon_{L2r} + i\epsilon_{L2i}) h_{\alpha r} \gamma_\alpha^2. \tag{2.22}$$

The notations used here are as follows (cf., NA):

$$\epsilon_{s0r} = 1 - W_a P_a, \tag{2.23}$$

$$\epsilon_{s0i} = V_a Q_a, \tag{2.24}$$

$$\epsilon_{s2r} = W_\alpha P_a / h_{\alpha r} + W_a P_2, \tag{2.25}$$

$$\epsilon_{s2i} = V_\alpha Q_a / h_{\alpha r} + V_a Q_2; \tag{2.26}$$

$$\epsilon_{L0r} = W_a \mathscr{P}_{La} P_a, \tag{2.27}$$

$$\epsilon_{L0i} = V_a \mathscr{Q}_{La} Q_a, \tag{2.28}$$

$$\epsilon_{L2r} = (W_\alpha \mathscr{P}_{La} + W_a \mathscr{P}_{L\alpha}) P_a / h_{\alpha r} + W_a \mathscr{P}_{La} P_2, \tag{2.29}$$

$$\epsilon_{L2i} = (V_\alpha \mathscr{Q}_{La} + V_a \mathscr{Q}_{L\alpha}) Q_a / h_{\alpha r} + V_a \mathscr{Q}_{La} Q_2; \tag{2.30}$$

$$W_a = W \quad (x = y = 0), \tag{2.31}$$

$$W_\alpha = \left(\frac{\partial W}{\partial \gamma_\alpha^2} \right)_{(x = y = 0)}, \tag{2.32}$$

and similarly for W replaced by $V, \mathscr{P}_L, \mathscr{Q}_L$;

$$P_a = P(x = y = 0)$$
$$= P(I = I_a), \tag{2.33}$$

$$P_2 = \left(\frac{\partial P}{\partial \gamma_\alpha^2} \right)_{(x = y = 0)} / h_{\alpha r}$$
$$= -I_a \left(\frac{\partial P_a}{\partial I_a} \right), \tag{2.34}$$

and similarly for P replaced by Q.

III. EQUATION FOR THE AVERAGE FIELD

When $k_0^2 \gg |\nabla^2 \ln \epsilon|$, the electric field of a plane-polarized laser beam in a plasma is governed by a scalar wave equation.[5] It is more convenient to write this equation in the form adopted in NA, as follows:

$$\frac{\partial^2 E}{\partial \mathscr{Z}^2} + q \sum_\alpha \frac{\partial^2 E}{\partial \gamma_\alpha^2} + q^2 \epsilon E = 0, \tag{3.1}$$

$$\mathscr{Z} \equiv z/k_0 r_0^2, \tag{3.2}$$

$$\gamma_\alpha \equiv \alpha/r_0 \quad (\text{for } \alpha = x \text{ and } y), \tag{3.3}$$

$$q \equiv k_0^2 r_0^2, \tag{3.4}$$

$$k_0 = \omega/c. \tag{3.5}$$

The boundary condition for Eq. (3.1) is given by

$$E(\mathscr{E}=0)=E_0\exp\Big(i\omega t-\sum_\alpha \gamma_\alpha^2/\alpha_0^2\Big), \tag{3.6}$$

where $\alpha_0(=x_0$ and $y_0)$ is the initial beamwidth (along the x and y axes, respectively).

In terms of the complex-valued amplitude envelope A and the normalized wavenumber

$$k=\epsilon_{or}^{1/2}, \tag{3.7}$$

the solution of Eq. (3.1) is written as

$$E=A\exp\Big(i\omega t-iq\int_0^{\mathscr{E}} kd\mathscr{E}\Big). \tag{3.8}$$

It has been assumed here that $\epsilon_{oi}\ll\epsilon_{or}$. In the WKBJ approximation, A is governed by

$$-2iqk\frac{\partial A}{\partial \mathscr{E}}+q\sum_\alpha\frac{\partial^2 A}{\partial\gamma_\alpha^2}-iq\frac{dk}{d\mathscr{E}}A$$
$$-q^2\Big[i\epsilon_{oi}+\sum_\alpha(\epsilon_{\alpha r}+i\epsilon_{\alpha i})h_{\alpha r}\gamma_\alpha^2\Big]A$$
$$-q^2\epsilon_tA=0. \tag{3.9}$$

Ensemble average[1,6] of this equation gives

$$-2iqk\frac{\partial\mathscr{A}}{\partial\mathscr{E}}+q\sum_\alpha\frac{\partial^2\mathscr{A}}{\partial\gamma_\alpha^2}-iq\frac{dk}{d\mathscr{E}}\mathscr{A}$$
$$-q^2\Big[i\epsilon_{oi}+\sum_\alpha(\epsilon_{\alpha r}+i\epsilon_{\alpha i})h_{\alpha r}\gamma_\alpha^2\Big]\mathscr{A}$$
$$-q^2\langle\epsilon_tA\rangle=0, \tag{3.10}$$

where

$$\mathscr{A}=\langle A\rangle. \tag{3.11}$$

Unless stated otherwise, the position coordinates of a quantity will be taken to be $\mathscr{E}$ and γ, where $\gamma=(\gamma_x,\gamma_y)$.

It will be assumed that the turbulence is microscopic as compared to the inherent or induced nonuniformity of the plasma. Then a local analysis regarding the effect of turbulence is justified. If the fluctuation ϵ_t is locally a Gaussian random variable, the correlation function $\langle\epsilon_t(\mathscr{E},\gamma)\epsilon_t(\mathscr{E}',\gamma')\rangle$ suffices to describe all the properties of ϵ_t and locally depends on the differences $\mathscr{E}-\mathscr{E}'$ and $\gamma-\gamma'$. Then the average $\langle\epsilon_tA\rangle$ may be expressed by the Novikov-Furutsu formula[6]

$$\langle\epsilon_tA\rangle=\int\int\int\langle\epsilon_t(\mathscr{E},\gamma)\epsilon_t(\mathscr{E}',\gamma')\rangle$$
$$\times\Big\langle\frac{\delta A(\mathscr{E},\gamma)}{\delta\epsilon_t(\mathscr{E}',\gamma')}\Big\rangle d\mathscr{E}'d\gamma_x'd\gamma_y', \tag{3.12}$$

where $\delta A/\delta\epsilon_t'$ denotes the functional derivative of A with respect to ϵ_t'. If the fluctuation ϵ_t is assumed to be delta-correlated along the direction of propagation, then

$$\langle\epsilon_t(\mathscr{E},\gamma)\epsilon_t(\mathscr{E}',\gamma')\rangle$$
$$=\delta(\mathscr{E}-\mathscr{E}')B(\mathscr{E},\tfrac{1}{2}(\gamma+\gamma');\gamma-\gamma'). \tag{3.13}$$

The presence of $\mathscr{E}$ and $\frac{1}{2}(\gamma+\gamma')$ in the arguments of B characterizes the local approximation employed here. Using Eq. (3.9), neglecting the backscattering induced by the fluctuation ϵ_t, and assuming that the self-focusing nonlinearity does not lead to any qualitative changes in the form of the turbulence, it can be shown[6] that

$$\frac{\delta A(\mathscr{E},\gamma)}{\delta\epsilon_t(\mathscr{E}',\gamma')}=\frac{iq}{4k}\delta(\gamma-\gamma')A(\mathscr{E},\gamma). \tag{3.14}$$

The Novikov-Furutsu formula (3.12) now reduces to

$$\langle\epsilon_tA\rangle=(iqB_0/4k)\mathscr{A}, \tag{3.15}$$

where

$$B_0=B(\mathscr{E},\gamma;\gamma-\gamma'=0). \tag{3.16}$$

Equation (3.11), with $\langle\epsilon_tA\rangle$ substituted from the expression (3.15), becomes

$$-2iqk\frac{\partial\mathscr{A}}{\partial\mathscr{E}}+q\sum_\alpha\frac{\partial^2\mathscr{A}}{\partial\gamma_\alpha^2}-iq\frac{dk}{d\mathscr{E}}\mathscr{A}$$
$$-q^2\Big[i\epsilon_{oi}+\sum_\alpha(\epsilon_{\alpha r}+i\epsilon_{\alpha i})h_{\alpha r}\gamma_\alpha^2$$
$$+\frac{iqB_0}{4k}\Big]\mathscr{A}=0. \tag{3.17}$$

In the paraxial region [characterized by Eq. (2.20)], B_0 may be expressed as

$$B_0=(B_{0or}+iB_{0oi})+\sum_\alpha(B_{0\alpha r}+iB_{0\alpha i})h_{\alpha r}\gamma_\alpha^2, \tag{3.18}$$

where

$$B_{0or}=W_a^2\langle\mathscr{P}_{1a}^2\rangle P_a^2-V_a^2\langle\mathscr{Q}_{1a}^2\rangle Q_a^2, \tag{3.19}$$
$$B_{0oi}=2W_aV_a\langle\mathscr{P}_{1a}\mathscr{Q}_{1a}\rangle P_aQ_a, \tag{3.20}$$
$$B_{0\alpha r}=W_a(W_a\langle\mathscr{P}_{1a}^2\rangle+W_a\langle\mathscr{P}_{1a}\mathscr{P}_{1a}\rangle)P_a^2/h_{\alpha r}$$
$$-V_a(V_a\langle\mathscr{Q}_{1a}^2\rangle+V_a\langle\mathscr{Q}_{1a}\mathscr{Q}_{1a}\rangle)Q_a^2/h_{\alpha r}$$
$$+W_a^2\langle\mathscr{P}_{1a}^2\rangle P_aP_\alpha-V_a^2\langle\mathscr{Q}_{1a}^2\rangle Q_aQ_\alpha, \tag{3.21}$$
$$B_{0\alpha i}=[(W_aV_a+W_aV_a)\langle\mathscr{P}_{1a}\mathscr{Q}_{1a}\rangle$$
$$+W_aV_a(\langle\mathscr{P}_{1a}\mathscr{Q}_{1a}\rangle+\langle\mathscr{P}_{1a}\mathscr{Q}_{1a}\rangle)]P_aQ_a/h_{\alpha r}$$
$$+W_aV_a\langle\mathscr{P}_{1a}\mathscr{Q}_{1a}\rangle(P_aQ_\alpha+P_\alpha Q_a). \tag{3.22}$$

Using the expression (3.18), Eq. (3.17) may be written as

$$-2iqk\frac{\partial\mathscr{A}}{\partial\mathscr{E}}+q\sum_\alpha\frac{\partial^2\mathscr{A}}{\partial\gamma_\alpha^2}-iq\frac{dk}{d\mathscr{E}}\mathscr{A}+\frac{q^3B_{0or}}{4k}\mathscr{A}$$
$$-q^2\Big[i\epsilon_{oti}+\sum_\alpha(\epsilon_{\alpha tr}+i\epsilon_{\alpha ti})h_{\alpha r}\gamma_\alpha^2\Big]\mathscr{A}=0. \tag{3.23}$$

The effective components of the dielectric constant introduced here are defined as

$$\epsilon_{oti}=\epsilon_{oi}+qB_{0or}/4k, \tag{3.24}$$
$$\epsilon_{\alpha tr}=\epsilon_{\alpha r}-qB_{0\alpha i}/4k, \tag{3.25}$$
$$\epsilon_{\alpha ti}=\epsilon_{\alpha i}+qB_{0\alpha r}/4k. \tag{3.26}$$

IV. SOLUTION FOR THE AVERAGE FIELD

Equation (3.23) is similar to its counterpart in NA. This similarity suggests that its solution may be expressed as

$$\mathscr{A}=\mathscr{A}_0\exp\Big(iq-\tfrac{1}{2}\sum_\alpha h_\alpha\gamma_\alpha^2\Big), \tag{4.1}$$

where

$$g = g_r + ig_i, \tag{4.2}$$

$$h_\alpha = h_{\alpha r} + ih_{\alpha i}. \tag{4.3}$$

Following the technique used in NA, it can be shown that g_r is given by

$$g_r = \ln\left(\frac{x_0 y_0 k(\mathscr{E}=0)}{r_0^2 f_x f_y k}\right) - q\int_0^{\mathscr{E}} \Big(\epsilon_{eai} + \tfrac{1}{2}\sum_\alpha \epsilon_{e\alpha i}\Big)k^{-1}\, d\mathscr{E}', \tag{4.4}$$

and that $h_{\alpha r}$ obeys the second-order ordinary differential equations

$$\frac{d^2 h_{\alpha r}}{d\mathscr{E}^2} - \frac{3k}{2h_{\alpha r}}\left(\frac{dh_{\alpha r}}{d\mathscr{E}}\right)^2 + \left(\frac{1}{k}\frac{dk}{d\mathscr{E}} + \frac{q\epsilon_{eai}}{k}\right)\frac{dh_{\alpha r}}{d\mathscr{E}} + \left(\frac{2h_{\alpha r}^3}{k^2} - \frac{2q\epsilon_{ear}h_{\alpha r}^2}{k^2} - \frac{q^2\epsilon_{eai}^2 h_{\alpha r}}{2k^2} - \frac{qh_{\alpha r}}{k}\frac{d\epsilon_{eai}}{d\mathscr{E}}\right) = 0. \tag{4.5}$$

The following boundary conditions have been assumed in writing the expression (4.4):

$$f_\alpha(\mathscr{E}=0) = a_0/r_0, \tag{4.6}$$

$$(df_\alpha/d\mathscr{E})_{\mathscr{E}=0} = 0. \tag{4.7}$$

Written in terms of the beamwidth parameter f_α, Eqs. (4.5) become

$$\frac{d^2 f_\alpha}{d\mathscr{E}^2} + \left(\frac{1}{k}\frac{dk}{d\mathscr{E}} + \frac{q\epsilon_{eai}}{k}\right)\frac{df_\alpha}{d\mathscr{E}} = \left(\frac{1}{k^2 f_\alpha^3} - \frac{q\epsilon_{ear}}{k^2 f_\alpha} - \frac{q^2\epsilon_{eai}^2 f_\alpha}{4k^2} - \frac{qf_\alpha}{2k}\frac{d\epsilon_{eai}}{d\mathscr{E}}\right). \tag{4.8}$$

Being nonlinear, Eqs. (4.5) or (4.8) have to be solved along with a simultaneous evaluation of the axial intensity I_a, which is given by

$$I_a = I_0 \exp g_r; \tag{4.9}$$

$$I_0 = \beta E_0^2. \tag{4.10}$$

From the expressions (3.20), (3.22), and (4.4), it is clear that the plasma turbulence tends to damp[1-6] the field intensity. The presence of the self-focusing nonlinearity complicates the manner in which this damping takes place; thus, unlike in a linear case, the field intensity in the present case may not damp monotonically with the propagation distance $\mathscr{E}$. The $\mathscr{E}$ dependence of the axial intensity I_a is self-consistently coupled with the $\mathscr{E}$ dependence of the beamwidth parameter f_α. In the case of a radially Gaussian laser beam, $f_\alpha = f$, the beamwidth parameter f is governed by Eq. (4.8). The behavior of f has already been discussed in detail in NA. In the case of an elliptically Gaussian[10] beam $[f_x(\mathscr{E}=0) \neq f_y(\mathscr{E}=0)]$, the beamwidth parameters f_α vary differently, but their variations with $\mathscr{E}$ are coupled to each other. This difference in the variations of f_α (for $\alpha = x$ and y) implies that the ellipticity of the laser beam cross section varies with the propagation distance $\mathscr{E}$.

V. NUMERICAL RESULTS

In order to understand the quantitative behavior of the average field of a laser beam self-focused in a turbulent plasma, Eqs. (4.8) have been solved by using the Runge-Kutta method.[11] Instead of adopting the sets of parameters from some particular experimental investigtions, they have been so chosen that they can illustrate the conclusions of the present investigation point by point. However, care has been taken to select only those parameters which can be realized in typical experiments.

Results of the numerical analysis carried out have been presented in Figs. 1–5 in the form of graphs of f (or f_α) and g_r versus $\mathscr{E}$. Figure 1 (for which $I_0 = 0$ and q = arbitrary) illustrates the effect of turbulence on the linear propagation; Fig. 2 illustrates the effect of turbulence on the self-focusing phenomenon in an ideal case; Fig. 3 (for which $V = 0.000\,08$) illustrates the effect of turbulence on self-focusing in an absorbing plasma, Fig. 4 [for which $W = 0.004(1 + 0.4\mathscr{E}^2)$] illustrates the effect of turbulence on self-focusing in an inhomogeneous plasma; and Fig. 5 (for which $2f_x(\mathscr{E}=0) = f_y(\mathscr{E}=0)/2 = 1$) illustrates the effect of turbulence on self-focusing of an elliptical beam. Except for the parameters mentioned in brackets above, the parameters have been fixed as follows: $I_0 = 0.2$, $q = 2500$, $V = 0$, $W = 0.004$, and $f_\alpha(\mathscr{E}=0) = f(\mathscr{E}=0) = 1$. In each figure, there are four graphs corresponding to four different levels of the plasma turbulence. $\langle \mathscr{P}_i^2 \rangle = 0$ for the graphs labelled A, 0.005 for B, 0.01 for C, and 0.02 for D. $\langle \mathscr{Q}_i^2 \rangle$ and $\langle \mathscr{P}_i$

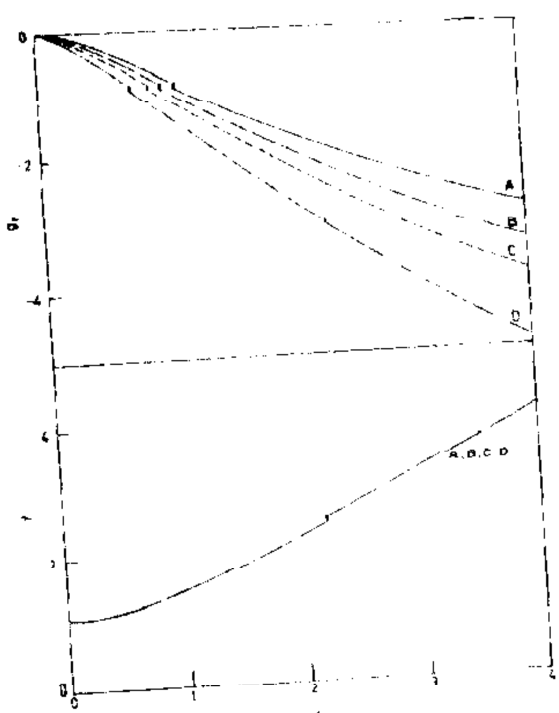

FIG. 1. Effect of turbulence on the linear propagation. $I_0 = 0$; q = arbitrary; $V = 0$; $W = 0.004$; $f_x(\mathscr{E}=0) = f(\mathscr{E}=0) = 1$, $\langle \mathscr{P}_i^2 \rangle$ = (A) 0, (B) 0.005, (C) 0.01, and (D) 0.02, $\langle \mathscr{Q}_i^2 \rangle$ = arbitrary, $\langle \mathscr{P}_i \mathscr{Q}_i \rangle$ = arbitrary

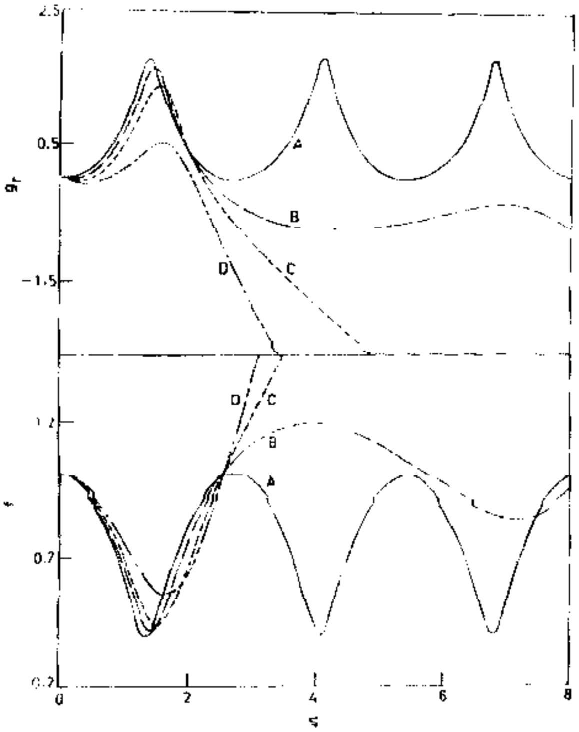

FIG. 2. Effect of turbulence on the self-focusing phenomenon. $I_0 = 0.2$; $q = 2500$; $V = 0$; $W = 0.004$; $f_0(\mathscr{Z} = 0) = f(\mathscr{Z} = 0) = 1$, $\langle \mathscr{P}_I^2 \rangle$ = (A) 0, (B) 0.005, (C) 0.01, and (D) 0.02; $\langle \mathscr{Q}_I^2 \rangle$ = arbitrary; $\langle \mathscr{P}_I \mathscr{Q}_I \rangle$ = arbitrary.

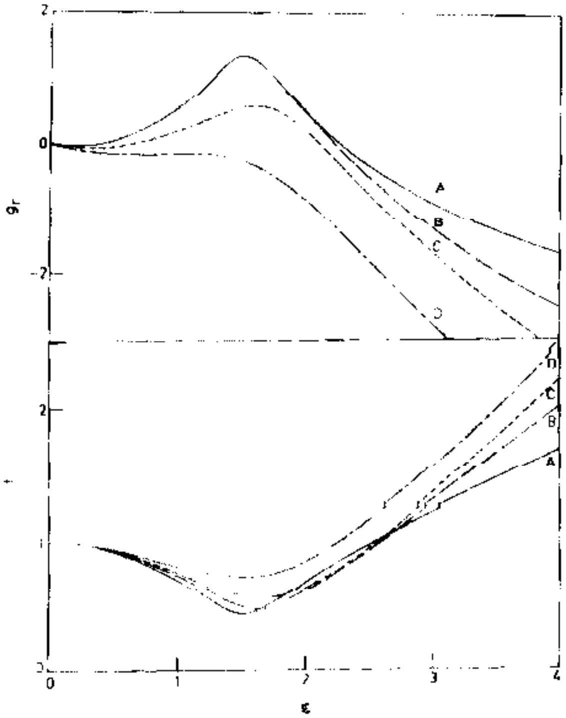

FIG. 3. Effect of turbulence on self-focusing in a nonlinearly absorbing plasma. $I_0 = 0.2$; $q = 2500$; $V = 0.000\,08$; $W = 0.004$; $f_0(\mathscr{Z} = 0) = f(\mathscr{Z} = 0) = 1$; $\langle \mathscr{P}_I^2 \rangle$ = (A) 0, (B) 0.005, (C) 0.01, (D) 0.02; $\langle \mathscr{Q}_I^2 \rangle$ = (A) 0, (B) 0.005, (C) 0.04, (D) 0.02; $\langle \mathscr{P}_I \mathscr{Q}_I \rangle$ = (A) 0, (B) 0.005, (C) 0.02, (D) 0.02.

$\mathscr{Q}_L \rangle$ are arbitrary except in Fig. 3, in which $\langle \mathscr{Q}_L^2 \rangle = \langle \mathscr{P}_L \mathscr{Q}_L \rangle = 0$ for A, $\langle \mathscr{Q}_L^2 \rangle = \langle \mathscr{P}_L \mathscr{Q}_L \rangle = 0.005$ for B, $\frac{1}{4}\langle \mathscr{Q}_L^2 \rangle = \frac{1}{2}\langle \mathscr{P}_L \mathscr{Q}_L \rangle = 0.01$ for C, and $\langle \mathscr{Q}_L^2 \rangle = \langle \mathscr{P}_L \mathscr{Q}_L \rangle = 0.02$ for D.

The main conclusions drawn from the numerical results presented in Figs. 1–5 are as follows:

(1) In the case of linear propagation, the beamwidth parameter (axial intensity) increases (decreases) monotonically with the propagation distance.

(2) In the case of linear propagation, the plasma turbulence does not affect the beamwidth parameter, but reduces the axial intensity of the laser beam.

(3) For the initial axial intensity I_0 chosen inbetween the lower and upper self-trapping intensities (see NA), the laser beam undergoes periodic focusing in a quiescent ideal plasma.

(4) The periodicity in the oscillations of the beamwidth parameter and axial intensity is destroyed by the presence of turbulence, absorption, or axial inhomogeneity. After a finite number of aperiodic oscillations, the beamwidth parameter (axial intensity) starts increasing (decreasing) monotonically.

(5) The number of aperiodic oscillations, after which the beamwidth parameter starts increasing monotonically, decreases with the level of turbulence, absorption, or axial inhomogeneity. For a sufficiently high level, the monotonic variation may start from the very beginning.

(6) If the beamwidth parameter undergoes oscillatory focusing for some initial distance of propagation, the first focal length (i.e., the minimum distance at which the beamwidth parameter attains a local minimum value) and the value of the beamwidth parameter at the first focal length are increased by the plasma turbulence or by absorption of the laser radiation. On the other hand, they are decreased by the axial inhomogeneity having $dW/d\mathscr{Z} > 0$.

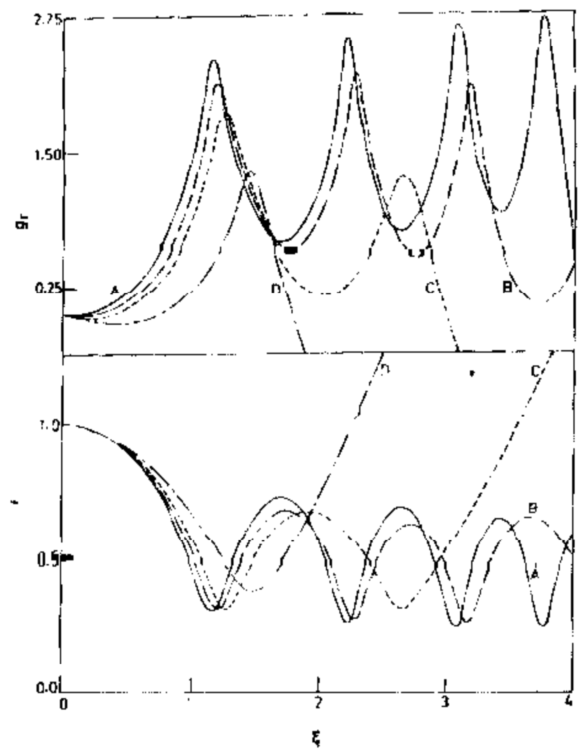

FIG. 4. Effect of turbulence on self-focusing in an axially inhomogeneous plasma. $I_0 = 0.2$, $q = 2500$; $V = 0$; $W = 0.004(1 + 0.4\mathscr{Z})$; $f_0(\mathscr{Z} = 0) = f(\mathscr{Z} = 0) = 1$; $\langle \mathscr{P}_I^2 \rangle$ = (A) 0, (B) 0.005, (C) 0.01, and (D) 0.02; $\langle \mathscr{Q}_I^2 \rangle$ = arbitrary; $\langle \mathscr{P}_I \mathscr{Q}_I \rangle$ = arbitrary.

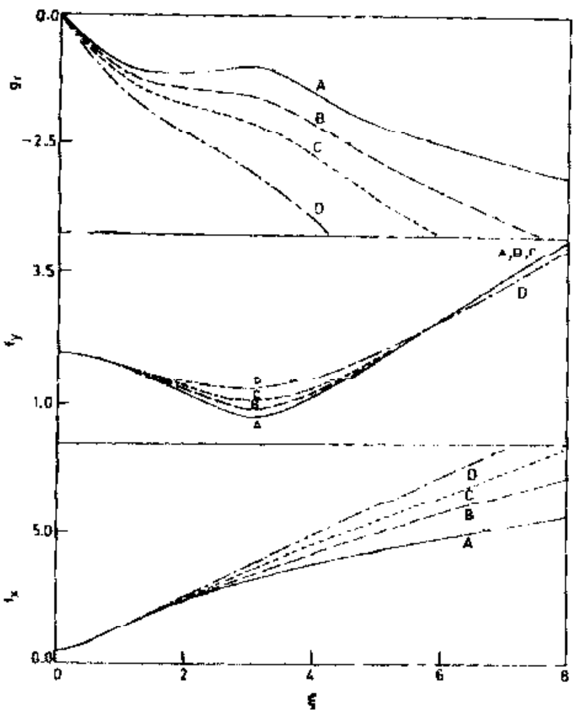

FIG. 5. Effect of turbulence on self-focusing of an elliptical beam. $I_0 = 0.2$; $q = 2500$; $V = 0$; $W = 0.004$; $2f_x(\mathscr{E} = 0) = f_y(\mathscr{E} = 0)/2 = 1$; $\langle \mu_1^2 \rangle$ = (A) 0, (B) 0.005, (C) 0.01, (D) 0.02; $\langle \mu_2^2 \rangle$ = arbitrary; $\langle \mu_1, \mu_2 \rangle$ = arbitrary.

(7) If the beamwidth parameter undergoes oscillatory focusing, extremes of the beamwidth parameter (axial intensity) go on decreasing (increasing) with the propagation distance in the presence of axial inhomogeneity having $dW/d\mathscr{E} > 0$. This characteristic may not be evident in the presence of turbulence or absorption.

(8) Like the collision-induced absorption, the plasma turbulence tends to reduce the field intensity as the laser beam penetrates further.

(9) Unless the level of turbulence is sufficiently high, the effect of turbulence is generally not so evident for some initial distance of propagation.

(10) If the laser beam cross section is elliptical (i.e., the initial values of the beamwidth parameters along the x and y axes are different), then there is a remarkable difference between the variations of the two beamwidth parameters (and consequently the ellipticity[11] of the laser beam cross section varies with the distance of propagation). In the case considered in Fig. 5, for example, f_x starts increasing monotonically from the very beginning, whereas f_y decreases before starting to increase monotonically.

(11) In the absence of turbulence, the existence of a local minimum of any of the two beamwidth parameters implies the existence of a local maximum of the axial intensity. This may not be so in the presence of turbulence.

(12) For $f_x(\mathscr{E} = 0) < f_y(\mathscr{E} = 0)$, f_x increases, whereas f_y decreases with the level of turbulence, for a given value of $\mathscr{E}$ in the region of monotonic rise of both the beamwidth parameters.

The analysis in the present investigation is in terms of normalized (dimensionless) quantities. This choice is not only convenient, but also theoretically more sound. A numerical analysis in terms of normalized quantities deals with the minimum number of initial data, and hence it has a broader scope than a corresponding numerical analysis in terms of absolute (dimensional) quantities. While correlating the present investigation with an experiment, it becomes necessary to convert the presented normalized parameters into the required absolute parameters. As an illustration, it may be necessary to find out a typical set of values of the absolute parameters which correspond to the set $I_0 = 0.2$, $q = 2500$, $W = 0.004$ of normalized parameters mentioned above. If $\omega = 10^{14}$/sec, and $T_0 = 10^7$ °K, then r_0 = initial beamwith $= q^{1/2}c/\omega = 0.015$ cm, $E_0^2 = 4mk_B T_0 \omega^2 I_0/e^2 = 4.5\times 10^{10}$ erg/cm^3, and $N_0 = W m\omega^2/4\pi e^2 = 1.2\times 10^{12}$/cm^3. Note that these values for the absolute parameters are realizable in an experimental setup.

In the graphs plotted in Figs. 1–5, the characteristic scale lengths of the variations of the beamwidth parameter and the axial intensity are larger than $\mathscr{E} = 0.1$. For the value (2500) of q chosen in the present numerical analysis, $\mathscr{E} = 0.1$ means $z = 250/k_0 = 250$ times the vacuum wavelength. Thus the WKBJ approximation employed in the present investigation is justified.

ACKNOWLEDGMENTS

The authors are grateful to Dr. R.P. Sharma and Mr. D. Subbarao for useful discussions and to the reviewer of this paper for many valuable comments. This work was partially supported by the Indian National Science Academy.

[1] *Laser Beam Propagation in the Atmosphere* edited by J.W. Strohbehn (Springer-Verlag, Berlin, 1978); R.L. Fante, Proc. IEEE **63**, 1669 (1975)

[2] *Laser Plasmas and Nuclear Energy*, edited by H. Hora (Plenum, New York, 1975).

[3] S.A. Akhmanov, R.V. Khokhlov, and A.P. Sukhorukov, in *Laser Handbook*, edited by F.T. Arecchi and E.O. Schulz Du Bois (North Holland, Amsterdam, 1972) p. 1151.

[4] O. Svelto, Prog. Opt. **12**, 1 (1974); Y.R. Shen, Prog. Quantum Electron. **4**, 1 (1975); J.H. Marburger, Prog. Quantum Electron. **4**, 35 (1975).

[5] M.S. Sodha, A.K. Ghatak, and V.K. Tripathi, Prog. Opt. **13**, 169 (1976); M.S. Sodha and V.K. Tripathi, Phys. Rev. A **16**, 2101 (1977).

[6] A. Ishimaru, *Wave Propagation and Scattering in Random Media* (Academic, New York, 1978), Vol. 2.

[7] M.S. Sodha, R.P. Sharma, and S.C. Kaushik, J. Appl. Phys. **46**, 4697 (1975); J.M. Chapman and J. Kevorkian, J. Appl. Phys. **49**, 4722 (1978).

[8] M.S. Sodha, L.A. Patel, and R.P. Sharma, J. Appl. Phys. **49**, 3707 (1978)

[9] M.S. Sodha, L.A. Patel, and S.C. Kaushik, Plasma Phys. **21**, 1 (1979).

[10] A.B. Shartsburg, Radio Phys. Quantum Electron. **13**, 1368 (1970); M.S. Sodha, R.P. Sharma, S. Kumar, and V.K. Tripathi, Opt. Acta **23**, 305 (1976).

[11] J.B. Scarborough, *Numerical Mathematical Analysis* (Johns Hopkins, Oxford, 1966) p. 361.