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Ion-acoustic solitons in an electromagnetically irradiated magnetoplasma

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We have investigated how the presence of an electromagnetic beam and a static magnetic field influences the ion-acoustic solitons in a plasma. A modified KdV equation is derived in which the electromagnetic field plays the role of a source term. By using the perturbation analysis technique, it is shown that a homogeneous electromagnetic beam does not destabilize the ion-acoustic solitons; it reduces the amplitude, but not the velocity, of the solitons. However, any inhomogeneity in the electromagnetic field intensity does destabilize the ion-acoustic solitons; albeit not to an appreciable extent for typical cases.

1. Introduction

The ion-acoustic solitons (Davidson 1972; Karpman 1975) have been widely discussed, but it is only recently that some studies have been made on important aspects like inherent two-dimensionality (Ogino & Takeda 1975), kinetics (Zakharov 1971), magnetically induced shape-distortion (Zakharov & Kuznetsov 1974), interaction with an electromagnetic beam (Kaw & Nishikawa 1975), effect of inhomogeneity (Nishikawa & Kaw 1975), effect of ion-temperature (Tagare 1975), and quantum field-theoretic formulation (Goldstone & Jackiw 1975). Experimental studies (Watanabe 1975) do indicate the significance of the concept of solitons in an understanding of the plasma turbulence (Ichimaru 1975).

In the present paper, we have investigated how the presence of an electromagnetic beam and a longitudinal static magnetic field influences the ion-acoustic solitons in a plasma. The investigation is based primarily on Zakharov & Kuznetsov's (1974) investigation on the magnetically induced shape-distortion and on Kaw & Nishikawa's (1975) investigation on the interaction with an electromagnetic beam.

In §2, we discuss the basic equations employed in the analysis of the problem. An equation is obtained for the ion velocity. In §3, we have considered the small velocity case investigated earlier by Kaw & Nishikawa (1975). The corresponding solution represents a propagating filamentary electromagnetic beam. In §4, we have reduced the forementioned equation to a modified KdV equation by assuming that the ion motion is confined to some specific direction. In this equation, the electromagnetic beam intensity plays the role of a source term. In §5, we employ the perturbation analysis technique to solve the modified KdV equation. We have assumed the static magnetic field to be small enough not to cause considerable transverse inhomogeneity in the ion-acoustic solitons, and the electromagnetic beam to be weak enough for the perturbation analysis to be valid. This analysis shows that the presence of a homogeneous electromagnetic field does not destabilize the ion-acoustic solitons; it reduces the amplitude, but not the velocity, of the solitons. On the other hand, any inhomogeneity in the electromagnetic beam-intensity destabilizes the ion-acoustic solitons; this destabilization is, however, not appreciable in typical cases.

In §6, we have considered the electromagnetic beam to be transversely Gaussian, so that it becomes self focused as it propagates through the plasma. We have quoted a simple analytic expression for the beam intensity by assuming the ponderomotive force on electrons to be the only mechanism for self-focusing. Then by assuming that the ion-acoustic solitons travel along the direction of propagation of the electromagnetic beam (i.e. the direction of the static magnetic field), we have analysed the perturbation in the solitons due to such a self-focused electromagnetic beam. In §7, the foregoing analytical results (based on the perturbation analysis) have been illustrated with some relevant graphs for typical parameters.

2. Basic equations

The electric field $\mathbf{E}(\mathbf{r}, t)$ of an electromagnetic beam propagating in a plasma obeys the wave equation

$$[\partial_t^2 - c^2\nabla^2 + \omega_p^2]\,\mathbf{E} = 0, \tag{2.1}$$

where the square of the local plasma frequency

$$\begin{aligned}\omega_p^2(\mathbf{r}, t) &= 4\pi n_e(\mathbf{r}, t)\, e^2/m = \omega_{p0}^2\, n_e(\mathbf{r}, t)/n_0 \\ &= \omega_{p0}^2(1 + \delta n_e(\mathbf{r}, t)/n_0) = \omega_{p0}^2 + \delta\omega_p^2(\mathbf{r}, t).\end{aligned} \tag{2.2}$$

Let us take this electric field to be of the form

$$\mathbf{E}(\mathbf{r}, t) = \hat{x}\mathscr{E}(\mathbf{r}, t) \exp i[kz - \omega t + \theta(\mathbf{r}, t)], \tag{2.3}$$

where the field envelope $\mathscr{E}(\mathbf{r}, t)$ and the phase $\theta(\mathbf{r}, t)$ are real and obey the coupled equations

$$[\partial_t^2 - c^2\nabla^2 + 2\omega(\partial_t\theta) + 2c^2k(\partial_t\theta) - (\partial_t\theta)^2 + c^2(\nabla\theta)^2 + \delta\omega_p^2]\,\mathscr{E} = 0, \tag{2.4}$$

$$[(\partial_t^2\theta) - c^2(\nabla^2\theta) - 2\{\omega\partial_t + c^2k\partial_z - (\partial_t\theta)\,\partial_t + c^2(\nabla\theta)\,.\,\nabla\}]\,\mathscr{E} = 0, \tag{2.5}$$

and where the linear wavenumber

$$k = (\omega^2 - \omega_{p0}^2)^{\frac{1}{2}}/c. \tag{2.6}$$

We shall assume the ions of the plasma to be non-relativistic so that the magnetic field associated with the electromagnetic beam will be neglected. However, we shall include the contribution from the static magnetic field $\mathbf{H}_0$ in the following treatment. The net electric field, i.e. the electric field due to the beam plus the density perturbations, will be represented as usual by $(-\nabla\phi)$.

Let us neglect the electron inertia and use the isothermal equation of state $P_e = n_e \kappa T_e$ where κ is the Boltzmann constant. We then have, in the quasi-steady state, the force equation (Kaw & Nishikawa 1975)

$$e\nabla\phi - \kappa T_e \nabla(\ln n_e) - (e^2/2m\omega^2)\nabla\mathscr{E}^2 = 0. \tag{2.7}$$

This, upon integration, yields the Boltzmann distribution

$$n_e = n_0 \exp\left[e\phi/\kappa T_e - \mathscr{E}^2 e^2/2m\omega^2\kappa T_e\right]. \tag{2.8}$$

(Deviation from the assumed isothermality of the electrons will introduce (Tagare 1975) half-integral powers of ϕ in the power series expansion of n_e.) Poisson's equation then becomes

$$\nabla^2\phi = 4\pi e\{n_0 \exp\left[e\phi/\kappa T_e - \mathscr{E}^2 e^2/2m\omega^2\kappa T_e\right] - n\}. \tag{2.9}$$

Moreover, we have the continuity, momentum and pressure equations (Tagare 1975),

$$\partial_t n + \nabla.(n\mathbf{V}) = 0, \tag{2.10}$$

$$\partial_t \mathbf{V} + (\mathbf{V}.\nabla)\mathbf{V} = -(e/M)\nabla\phi + \mathbf{V}\times\boldsymbol{\omega}_{H0} - (T/nT_{ef})\nabla p, \tag{2.11}$$

and

$$\partial_t p + (\mathbf{V}.\nabla)p + \gamma p\nabla.\mathbf{V} = 0. \tag{2.12}$$

Here $n(\mathbf{r},t)$, $\mathbf{V}(\mathbf{r},t)$, M, $\boldsymbol{\omega}_{H0} \equiv \mathbf{H}_0 e/Mc$, T and $p(\mathbf{r},t)$ are respectively the number density, velocity, mass, cyclotron frequency, temperature and pressure of the ions. In the present case of isothermal electrons, the free electron temperature T_{ef} is the same as T_e. In (2.12),

$$\gamma \equiv p/n\kappa T = 3$$

for adiabatic ion motion.

For long wavelength [$\lambda \gg$ Debye length $\lambda_D \equiv (\kappa T_e/4\pi n_0 e^2)^{\frac{1}{2}}$] and weak nonlinearity ($\delta n_e \ll n_0$), (2.9) yields

$$\frac{e\phi}{\kappa T_e} \simeq (1+\lambda_D^2\nabla^2)\left[\frac{\mathscr{E}^2 e^2}{2m\omega^2\kappa T_e} + \frac{\delta n_e}{n_0} - \frac{1}{2}\left(\frac{\delta n_e}{n_0}\right)^2\right]. \tag{2.13}$$

We shall assume the ion fluid to be cold so that $\nabla p = 0$ and (2.12) is then not required. Eliminating ϕ from (2.11) by using (2.13), we get

$$\partial_t \mathbf{V} + \nabla\left\{(1+\lambda_D^2\nabla^2)\left[\frac{\mathscr{E}^2 e^2}{2Mm\omega^2} + \frac{\kappa T_e}{M}\frac{\delta n_e}{n_0} - \frac{\kappa T_e}{2M}\left(\frac{\delta n_e}{n_0}\right)^2\right] - \frac{V_0^2}{2}\right\} = \mathbf{V}\times\boldsymbol{\omega}_{H0}. \tag{2.14}$$

Note that this equation does not have a 'divergence form'. It therefore implies that the ion momentum is, in general, not conserved. Consequently, unlike the case of no static magnetic field, a soliton solution can exist only if the ion motion is assumed to be confined to some particular direction. Also note that (2.14) is coupled with (2.10), (2.4) and (2.5). It is necessary to get rid of this coupling between the equations, in order to make the problem solvable.

3. Small velocity case

Let us first consider a particular case investigated by Kaw & Nishikawa (1975). Let $H_0 = 0$. When the density perturbations are small (i.e. $n_e \simeq n_0$) and the field envelope $\mathscr{E}$ does not vary appreciably with time, one obtains (Kaw & Nishikawa 1975)

$$[\partial_t^2 - (\kappa T_e/M)\nabla^2]\,\delta n_e \simeq (\kappa T_e/M)\nabla^2\mathscr{E}^2. \tag{3.1}$$

This equation describes an ion wave propagating in a direction depending upon the ponderomotive force due to the electromagnetic beam. Let us look for the solitons propagating with a constant velocity U (with $U \ll c$) and introduce a new variable

$$\mathbf{R} = \mathbf{r} - \mathbf{U}t. \tag{3.2}$$

Equation (2.1) then gives

$$\delta n_e = -\mathscr{E}^2/(1 - U^2M/\kappa T_e), \tag{3.3}$$

and (2.5) yields, to the first order approximation,

$$\theta = \omega\mathbf{R}.\mathbf{U}/(c^2 - U^2). \tag{3.4}$$

Substituting (3.3) and (3.4) into (2.4), we obtain

$$\left\{\nabla^2 - \left(\frac{2\omega Uk - 2\omega^2U^2/c^2}{c^2 - U^2}\right) + \frac{\omega_{p0}^2\mathscr{E}^2}{n_0(1 - U^2M/\kappa T_e)(c^2 - U^2)}\right\}\mathscr{E} = 0. \tag{3.5}$$

Equation (3.5) is a type of nonlinear Schrödinger equation. Let us consider only the one-dimensional version of it (Kaw & Nishikawa 1975), and put

$$\nabla^2 = \partial_y^2, \quad \mathbf{U} = \hat{y}U \quad \text{and} \quad \mathbf{R} = \hat{y}Y = \hat{y}(y - Ut). \tag{3.6}$$

Then the solution of (3.5), under the boundary condition that the solution and its first derivative vanish at $R \to \pm\infty$, may be written as

$$\mathscr{E} = [2(2\omega Uk - 2\omega^2U^2/c^2)(1 - U^2M/\kappa T_e)(n_0/\omega_{p0}^2)]^{\frac{1}{2}} \times \operatorname{sech}\left[\left(\frac{2\omega Uk - 2\omega^2U^2/c^2}{c^2 - U^2}\right)^{\frac{1}{2}} Y\right]. \tag{3.7}$$

This solution represents a propagating filamentary electromagnetic beam.

4. Extended KdV equation

In order to simplify (2.14), let us assume that the waves described by this equation travel only in some specific direction, say $\hat{l}$. This arbitrary direction $\hat{l}$ may be the direction of the static magnetic field or the y axis depending upon whether $H_0^2 \gg \mathscr{E}^2$ or $H_0^2 \ll \mathscr{E}^2$. We shall moreover assume that the electric field intensity $\mathscr{E}^2$ is already known so that it need not be derived along with (2.14), and that $\lambda_D^2 \nabla^2 \mathscr{E}^2 \ll \mathscr{E}^2$. Following the arguments given by Zakharov & Kuznetsov (1974), it can be shown that

$$\partial_t V_l + c_s \partial_l \left\{ \frac{V_l^2}{2c_s} + \left[1 + \frac{\lambda_D^2 \partial_l^2}{2} + \frac{\lambda_H^2 + \lambda_D^2}{2} \nabla_{\perp l}^2 \right] V_l + \frac{\mathscr{E}^2 e^2 c_s}{2m\omega^2 \kappa T_e} \right\} = 0, \tag{4.1}$$

where the ion-sound speed $c_s \equiv (\kappa T_e/M)^{\frac{1}{2}}$ and

$$\lambda_H = \hat{l}.\hat{H}_0 c_s/\omega_0. \tag{4.2}$$

We shall now employ the following dimensionless variables which form a co-ordinate system moving with velocity $c_s \hat{l}$.

$$\xi = \frac{l - c_s t}{\lambda_D}, \quad \boldsymbol{\xi}_\perp = \frac{\mathbf{r} - \mathbf{r}.\hat{l}}{(\lambda_H^2 + \lambda_D^2)^{\frac{1}{2}}},$$

$$\tau = \frac{t\omega_{pi}}{2} = t(\pi n_0 e^2/M)^{\frac{1}{2}}, \quad u = \frac{V_l}{2c_s}$$

and

$$\gamma^2 = \frac{\mathscr{E}^2 e^2}{4m\omega^2 \kappa T_e}. \tag{4.3}$$

We can then write (4.1) in the dimensionless form

$$\partial_\tau u + \partial_\xi (u^2 + \nabla_{\boldsymbol{\xi}}^2 u + \gamma^2) = 0. \tag{4.4}$$

5. Perturbation analysis

No analytic solution of (4.4) exists. However, the KdV equation

$$\partial_\tau W + \partial_\xi (W^2 + \partial_\xi^2 W) = 0 \tag{5.1}$$

does have an analytic solution: the soliton solution

$$W = (3\mu/2)\,\mathrm{sech}^2\,[X\mu^{\frac{1}{2}}/2], \tag{5.2}$$

where $X \equiv (\xi - \mu\tau)$ and μ is a parameter determining the amplitude, velocity and width of the soliton. Let us assume that it is possible to transform (4.4) into (5.1). This is possible if (but not only if) the following conditions are satisfied.

$$\partial_\tau W = \partial_\tau u, \tag{5.3}$$

$$W = u + \psi, \tag{5.4}$$

and

$$\psi^2 + 2u\psi + \partial_\xi^2 \psi = \nabla_{\perp\xi}^2 u + \gamma^2. \tag{5.5}$$

In the first order approximation, let us assume that $\partial_\xi^2 \psi = \nabla_{\perp\xi}^2 u = 0$, so that (5.5), in conjunction with (5.3) and (5.4), gives

$$W = U_1 + \psi_1 = +(U_1^2 + \gamma^2)^{\frac{1}{2}}, \tag{5.6}$$

so that

$$U_1 = +(W^2 - \gamma^2)^{\frac{1}{2}}. \tag{5.7}$$

We shall assume that $\gamma^2 \ll U_1^2$, so that

$$\psi_1 \simeq \gamma^2/2W. \tag{5.8}$$

In the second-order approximation, we write (5.5) as

$$\psi_{r_2}^2 + 2U_2\psi_{r_2} = \Gamma^2, \tag{5.9}$$

where

$$\Gamma^2 = \gamma^2 + \nabla_{\perp\xi}^2 U_1 - \partial_\xi^2\psi_1$$
$$\simeq \gamma^2 + \nabla_{\perp\xi}^2(W^2-\gamma^2)^{\frac{1}{2}} - (\partial_\xi^2\gamma^2)/2W. \tag{5.10}$$

Equation (5.9) yields

$$U_2 = +(W^2-\Gamma^2)^{\frac{1}{2}}. \tag{5.11}$$

6. Self-focusing

In the presence of an inhomogeneous electromagnetic beam, the electrons of the plasma experience a ponderomotive force (Lindl & Kaw 1971). Consequently a beam whose intensity profile is radially decreasing becomes self-focused (Sodha *et al.* 1976). We shall consider the radial intensity profile of the beam to be Gaussian, and consider only the ponderomotive force on the electrons as the nonlinear mechanism influencing the intensity profile of the beam. It can be shown (Sodha *et al.* 1976) that γ^2 may be expressed in the form

$$\gamma^2 = \gamma_0^2 \exp(-\rho^2/\rho_0^2 f^2), \tag{6.1}$$

where

$$\gamma_0 = \gamma \text{ at } \mathbf{r} = 0, \quad \rho^2 = x^2+y^2,$$

ρ_0 = mean beam width at $z = 0$, and the beam width parameter $f(z)$, for $\rho < \rho_0$ and $Z < Z_f$, is given by

$$f^2 = 1 - Z^2/Z_f^2, \tag{6.2}$$

where

$$Z_f^2 = (\omega^2-\omega_{p0}^2)\rho_0^2[\omega_{p0}^2\gamma_0^2/2 - c^2/\rho_0^2]^{-1}. \tag{6.3}$$

We shall now assume that the ion-acoustic waves travel in the direction of propagation of the electromagnetic beam i.e. $\hat{l} = \hat{z}$. We shall take the static magnetic field $\mathbf{H}_0$ to be longitudinal i.e. $\mathbf{H}_0 = \hat{z}H_0$, so that

$$\lambda_H^2 = c_s^2/\omega_{H0}^2 = (c^2 M\kappa T_e/e^2H_0^2).$$

Hence

$$\nabla_{\perp\xi}^2(W^2-\gamma^2)^{\frac{1}{2}} = (\lambda_H^2+\lambda_D^2)\rho^{-1}\partial_\rho\rho\partial_\rho(W^2-\gamma^2)^{\frac{1}{2}}$$
$$= \frac{(\lambda_H^2+\lambda_D^2)\gamma^2}{\rho_0^2 f^2(W^2-\gamma^2)^{\frac{1}{2}}}\left[2 - \frac{2\rho^2}{\rho_0^2 f^2} - \frac{\rho^2\gamma^2}{\rho_0^2 f^2(W^2-\gamma^2)}\right], \tag{6.4}$$

and

$$\partial_\xi^2\gamma^2 - \lambda_D^2\partial_Z^2\gamma^2 - \frac{2\lambda_D^2\gamma^2\rho^2}{Z_f^2\rho_0^2 f^4}\left[\left(\frac{\rho^2}{\rho_0^2 f^2} \quad 2\right)\frac{2Z^2}{f^2 Z_f^2} - 1\right]. \tag{6.5}$$

7. Discussion

In the perturbation analysis in §5, we neglected the magnetically induced transverse inhomogeneity of the ion-acoustic solitons in the absence of the electromagnetic field. This type of inhomogeneity was studied earlier by Zakharov & Kuznetsov (1974) and was shown not to destabilize the ion-acoustic solitons. Our treatment is valid for the static magnetic field small enough to

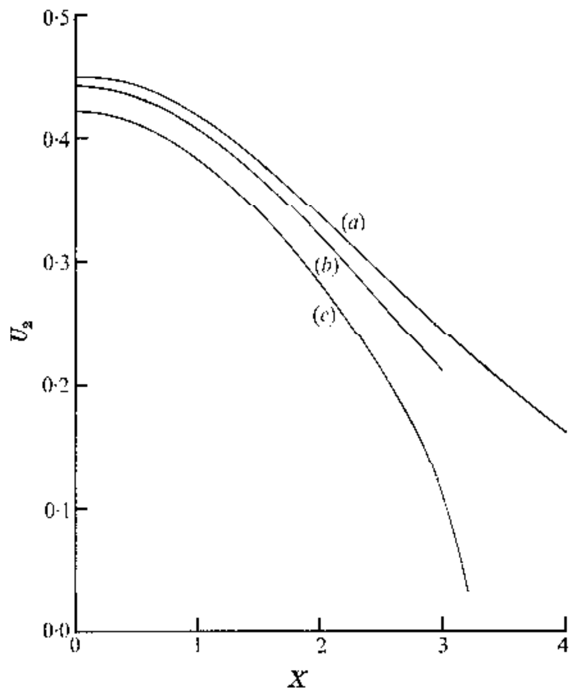

FIGURE 1. U_2 *versus* X $(= \xi - ut)$. (a) $\rho = 5$, $Z = 0$, (b) $\rho = 0$, $Z = 0$, (c) $\rho = 0$, $Z = 14$ (distances in centimetres).

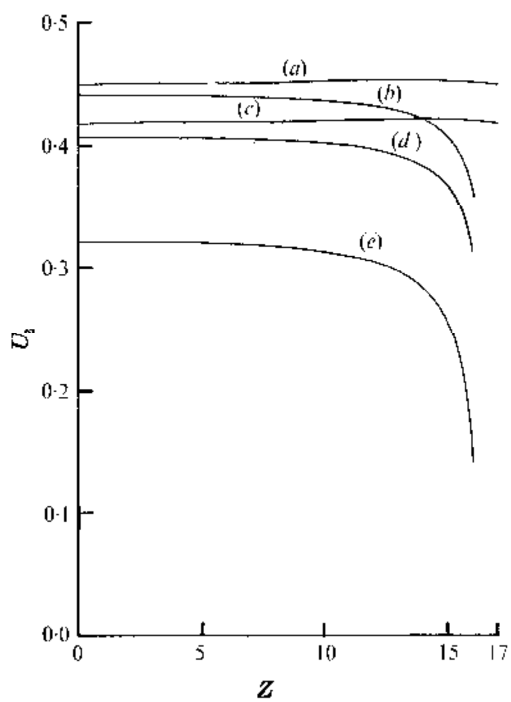

FIGURE 2. U_2 *versus* Z. (a) $\rho = 5$, $X = 0$, (b) $\rho = 0$, $X = 0$, (c) $\rho = 5$, $X = 1$, (d) $\rho = 0$, $X = 1$, (e) $\rho = 0$, $X = 2$.

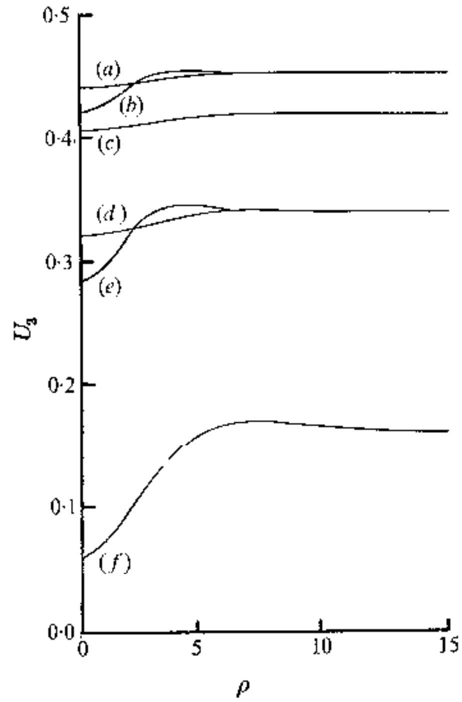

FIGURE 3. U_2 *versus* ρ. (a) $Z = 0, X = 0$, (b) $Z = 14, X = 0$, (c) $Z = 0, X = 1$, (d) $Z = 0, X = 2$, (e) $Z = 14, X = 2$, (f) $Z = 0, X = 4$.

allow us to take $\nabla^2_{\perp\xi} u_{(\gamma=0)} = 0$. Given this, we find from (5.7) that the presence of a homogeneous electromagnetic field does not destabilize the ion-acoustic solitons; it reduces the amplitude, but not the velocity, of the solitons. On the other hand, any inhomogeneity in the electromagnetic field intensity destabilizes the ion-acoustic solitons; the ion-acoustic waves are in the strictest sense no longer solitary waves, since they now depend not only on $X \equiv (\xi - \mu\tau)$ but also on ξ separately. A positive spatial gradient in the electromagnetic field intensity enhances the amplitude of the ion-acoustic solitons.

In order to illustrate the above analytical results, we have plotted, in figures 1, 2, and 3, the graphs of U_2 for the following parameters:

$$
\begin{aligned}
n_0 &= 10^{17}\,\mathrm{cm}^{-3},\\
T_e &= 10^4\,^\circ\mathrm{K},\\
H_0 &= 100\,\mathrm{Gauss},\\
M/m &= 2000,\\
\mu &= 0{\cdot}3,\\
\mathscr{E}_0^2 &= 10^4\,\mathrm{erg/cm^3},\\
\rho_0 &= 5\,\mathrm{cm},\\
\omega &= 10^{13}\,\mathrm{rad/sec}.
\end{aligned}
$$

Figures 1, 2 and 3 give U_2 *versus* X, Z and ρ respectively. These graphs, as expected, indicate that U_2 falls off at the focal point and increases with the radial distance. For the typical parameters we have chosen, we find, however, that, in most of the region, the solitons are not perturbed appreciably and hence the destabilization is not significant. This shows the inherent stability of the solitons against external perturbations.

We therefore conclude from the present investigation that the ion-acoustic solitons are reduced in amplitude in a homogeneous electromagnetic field and are destabilized in an inhomogeneous electromagnetic field in such a way that positive spatial gradients in the electromagnetic field intensity enhance the amplitudes of the solitons. In most of the region, under typical parameters, the solitons are not destabilized very much.

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