Quantum Entanglement Operator and Space-Time Variation

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Abstract

An entanglement operator matrix is introduced for the joint state of two quantum states. Entanglement between two particles attenuates as their distance or outward speed increases.

Introduction

Entanglement of quantum states [1,2,3] has been a subject of great dialog since early days of quantum physics. Its importance has now increased significantly because of its applications to quantum computing and quantum communication [4,5]. The existing formalism does not allow space-time variations of entanglement, and as a result, the subject of quantum teleportation [6,7,8] has generated a lot of debate. This paper introduces a formalism to allow space-time variations of entanglement. An entanglement operator matrix is introduced for the joint state of two quantum states. Entanglement between two particles attenuates as their distance or outward speed increases.

Entanglement operator

A quantum state, made of a linear superposition of two basis states |0> and |1>, is represented as

$$|\psi_1> = a_{11}\,|0> + a_{12}\,|1> = \begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} \tag{1}$$

Here a_{11} and a_{12} are complex probability amplitudes. Another similar quantum state is represented as

$$|\psi_2> = a_{21}\,|0> + a_{22}\,|1> = \begin{pmatrix} a_{21} \\ a_{22} \end{pmatrix} \tag{2}$$

Overlap of these two quantum states is

$$|< \psi_1|\psi_2 >|^2 = |(a_{11} \quad a_{12}) \begin{pmatrix} a_{21} \\ a_{22} \end{pmatrix}|^2 = |a_{11}a_{21} + a_{12}a_{22}\,|^2 \tag{3}$$

Under the existing framework of quantum mechanics, the joint state made of the above two quantum states is

$$\begin{aligned} |\psi_1, \psi_1 > &= |\psi_1 > |\psi_2 > = (a_{12}|0> + a_{12}\,|1>)(a_{21}\,|0> + a_{22}\,|1>) \\ &= a_{11}a_{21}\,|00> + a_{11}a_{22}\,|01> + a_{12}a_{21}\,|10> + a_{12}a_{22}\,|11> \\ &= \begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} (a_{21} \quad a_{22}) = \begin{pmatrix} a_{11}a_{21} & a_{11}a_{22} \\ a_{12}a_{21} & a_{12}a_{22} \end{pmatrix} \end{aligned} \tag{4}$$

It is hypothesized here that the joint state made of the quantum states $|\psi_1>$ and $|\psi_2>$ is

$$|\psi_1, \psi_1 > = \Theta.(|\psi_1 > |\psi_2 >) \tag{5}$$

Here Θ is the entanglement operator matrix.

$$\Theta = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} \tag{6}$$

Whereas $|\psi_i >$ defines individual characteristics of a quantum state, the entanglement operator matrix Θ defines jointness characteristics of the states on which it operates.

Using (5) and (6), the above joint state is

$$|\psi_1, \psi_1> = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} . (|\psi_1> |\psi_2>)$$
$$= \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} . \begin{pmatrix} a_{11}a_{21} & a_{11}a_{22} \\ a_{12}a_{21} & a_{12}a_{22} \end{pmatrix} = \begin{pmatrix} \theta_{11}a_{11}a_{21} & \theta_{12}a_{11}a_{22} \\ \theta_{21}a_{12}a_{21} & \theta_{22}a_{12}a_{22} \end{pmatrix} \tag{7}$$
$$= \theta_{11}\, a_{11}a_{21}\, |00> + \theta_{12}\, a_{11}a_{22}\, |01> + \theta_{21}\, a_{12}a_{21}\, |10> + \theta_{22}\, a_{12}a_{22}\, |11>$$

<u>Distance dependence</u>

In a case of no entanglement:

$$\Theta = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{8}$$

and

$$|\psi_1, \psi_1> = \begin{pmatrix} a_{11}a_{21} & a_{11}a_{22} \\ a_{12}a_{21} & a_{12}a_{22} \end{pmatrix} \tag{9}$$
$$= a_{11}a_{21}\, |00> + a_{11}a_{22}\, |01> + a_{12}a_{21}\, |10> + a_{12}a_{22}\, |11>$$

If normalization effects are ignored for simplicity, in a case of complete entanglement:

$$\Theta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{10}$$

and

$$|\psi_1, \psi_1> = \begin{pmatrix} a_{11}a_{21} & 0 \\ 0 & a_{12}a_{22} \end{pmatrix} \tag{11}$$
$$= a_{11}a_{21}\, |00> + a_{12}a_{22}\, |11>$$

which is a Bell state.

It is well known that $|\psi_i>$ varies in space-time according to forces acting on the state and these variations and forces can be represented by a Hamiltonian formalism. Likewise, Θ varies based on space-time points of its operands according to “entanglement forces”, which now need to be defined.

It is reasonable to assume that there is no entanglement between two particles when they are separated by a large distance (in space-time); their entanglement tends to increase as the distance between them decreases; and there is complete entanglement between them when they are in the closest possible proximity and other conditions are met.

It is also reasonable to assume that there is no entanglement between two particles when they are moving away from each other with a relative outward speed approaching to speed of light; their entanglement tends to increase as their relative outward speed decreases or relative inward speed increases; and there is complete entanglement between them when their relative inward speed approaches speed of light.

Under these assumptions, the entanglement operator matrix is

$$\Theta = \begin{pmatrix} 1 & e^{-(r_0/r)}\cos(\left(1+\frac{\mathrm{v}}{c}\right)\frac{\pi}{4}) \\ e^{-(r_0/r)}\cos(\left(1+\frac{\mathrm{v}}{c}\right)\frac{\pi}{4}) & 1 \end{pmatrix} \tag{12}$$

where r is distance between the two particles, r_0 is the "entanglement attenuation distance", v is the relative outward speed of the two particles, and c is speed of light.

Practical implications

According to equation (12), the entanglement between two particles attenuates as the distance between them increases. This implies that the possibility of teleportation decreases as the distance increases. This is understandable, since telecommunication engineers are well-versed in lossy communication channels.

A statistical extrapolation of equation (12) implies that chances of entanglement of particles increase as temperature decreases. This can explain why BCS pairing of particles takes place at low temperatures and not at high temperatures.

This formalism can also diffuse the EPR paradox [9], because the introduction of entanglement operator matrix prevents spooky action at a distance and teleportation at any distance.

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