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SELF-FOCUSING OF A LASER BEAM IN AN INHOMOGENEOUS PLASMA

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Abstract—This paper presents an analysis of self-focusing of a Gaussian laser beam in an axially inhomogeneous plasma. The nonlinearity in the dielectric constant arises on account of the carrier-redistribution caused by ponderomotive force or nonuniform heating of electrons with or without thermal conduction. It is seen that the laser beamwidth tends to attain a constant value depending upon the plasma inhomogeneity and initial laser intensity. Anomalous penetration of a laser beam has also been investigated in overdense plasmas with different types of inhomogeneities.

1. INTRODUCTION

IT IS WELL known (SODHA *et al.*, 1976) that a laser beam with radially nonuniform intensity-distribution can modify a plasma across the transverse cross-section. Such a modification leads to the phenomenon of self-focusing (AKHMANOV *et al.*, 1972; SODHA *et al.*, 1976) of a Gaussian laser beam. In a collisionless plasma, the laser self-focusing occurs on account of the carrier-redistribution caused by ponderomotive force (KAW *et al.*, 1973), whereas in a collisional plasma, it occurs on account of the carrier-redistribution caused by nonuniform heating of electrons (PRASAD and TRIPATHI, 1973). SODHA *et al.* (1973, 1975, 1976) and TOMLIJANOVICH (1975) have also investigated self-focusing in a thermally conducting collisional plasma in which nonuniform heating of electrons is limited by thermal conduction. Their investigations are based on perturbation approach and do not incorporate proper boundary conditions.

In this paper, we have analysed self-focusing of a Gaussian laser beam in an axially inhomogeneous plasma. In Section 2, the expressions for the dielectric constant have been presented when the carrier-redistribution is caused by ponderomotive force or nonuniform heating of electrons with or without thermal conduction. In Section 3, the wave equation for the electric field of the laser beam has been solved in the WKB and paraxial approximations (AKHMANOV *et al.*, 1972; SODHA *et al.*, 1976). In Section 4, numerical results along with a discussion have been presented.

It is seen that the temperature-dependence of the collision frequency (and consequently that of thermal conductivity) (GINZBURG, 1970; SHKAROFSKY *et al.*, 1966) and the boundary effects cannot be ignored when the nonuniform heating of electrons is limited by thermal conduction. In an axially inhomogeneous plasma, the laser beamwidth tends to attain a constant value depending upon the plasma-inhomogeneity and initial laser intensity. Anomalous penetration (ISALYEV *et al.*, 1976; SODHA and TRIPATHI, 1977) of a laser beam in an overdense plasma (whose plasma frequency exceeds the laser frequency) also seen to depend upon the plasma-inhomogeneity and initial laser intensity.

2. DIELECTRIC CONSTANT

We consider a Gaussian laser beam (propagating along the Z-axis) represented by

$$(\mathbf{E}^* \cdot \mathbf{E})_{(z=0)} = E_0^2 \exp(-r^2/r_0^2), \tag{2.1}$$

where r is the radial co-ordinate (of the cylindrical co-ordinate system) and r_0 is the initial laser beamwidth. If the beam diverges/converges slowly, the beam remains Gaussian (AKHMANOV *et al.*, 1972; SODHA *et al.*, 1976) and the intensity-distribution at Z is given by (c.f. Section 3)

$$(\mathbf{E}^* \cdot \mathbf{E}) = E_0{}^2 \varepsilon_{a(z=0)}^{1/2} \varepsilon_a^{-1/2} f^{-2} \exp(-r^2/r_0{}^2 f^2), \tag{2.2}$$

where $f(z)$ is the beamwidth parameter (AKHMANOV *et al.*, 1972; SODHA *et al.*, 1976) and $\varepsilon_a(z)$ is the dielectric constant on the axis.

On account of the laser-induced carrier-redistribution in a plasma, the electron concentration N in the presence of the laser beam differs from the original electron concentration $N_0(z)$ in the absence of the laser beam. In a collisionless plasma, the redistribution is caused by ponderomotive force (KAW *et al.*, 1973), and in the steady state (SODHA *et al.*, 1976), we have

$$N/N_0 = \exp(-\beta_p \mathbf{E}^* \cdot \mathbf{E}), \tag{2.3}$$

where

$$\beta_p = e^2/4m\kappa_B T_0 \omega^2, \tag{2.4}$$

e is the electron charge, m is the electron mass, κ_B is the Boltzmann constant, T_0 is the plasma temperature in the absence of the laser beam, and ω is the laser frequency. In a collisional plasma, in which the redistribution is caused by collision-dominated nonuniform heating of electrons, one obtains (SODHA *et al.*, 1976; c.f. Appendix)

$$N/N_0 = (1 + \beta_1 \mathbf{E}^* \cdot \mathbf{E})^{-1}, \tag{2.5}$$

where

$$\beta_1 = Me^2/6m^2 \kappa_B T_0 \omega^2, \tag{2.6}$$

M is the ion mass. When the redistribution, caused by nonuniform heating of electrons is limited by the thermal conduction, one obtains (c.f. Appendix)

$$N/N_0 = 2\{1 + [1 + 2sN_a{}^2 N_0{}^{-2} \varepsilon_{a(z=0)}^{1/2} \varepsilon_a^{-1/2} \cdot \beta_n E_0{}^2 (g - r^2/r_0{}^2 f^2)]^{1/s}\}^{-1}, \tag{2.7}$$

where

$$\beta_n = N_0 \nu_0 e^2 r_0{}^2 / 8\chi_0 T_0 m \omega^2, \tag{2.8}$$

N_a is the electron concentration on the axis, ν_0 and χ_0 are the electron–ion collision frequency and thermal conductivity of the plasma respectively in the absence of the laser beam,

$$s = 1 + \ln(\chi \nu_0 N_a / \chi_0 \nu N_0) / \ln(T/T_0) \tag{2.9}$$

is a parameter which characterizes the variation of the ratio χ/ν (of the thermal conductivity χ to the electron–ion collision frequency ν) with the electron temperature T, and

$$g = \int_0^{b^2/r_0{}^2} [1 - \exp(-x)] x^{-1} \, dx \tag{2.10}$$

is the boundary effect parameter determined by the boundary $r = b$ at which $T = T_0$.

The dielectric constant of a plasma (GINZBURG, 1970; SODHA *et al.*, 1976) is given by

$$\varepsilon = 1 - [\omega_p^2/(\omega^2 + i\omega\nu)](N/N_0), \tag{2.11}$$

where $\omega_p = (4\pi N_0 e^2/m)^{1/2}$ is the plasma frequency in the absence of the laser beam. When $\omega \gg \nu$, the imaginary part of the dielectric constant is negligible, and we may write

$$\varepsilon = 1 - (\omega_p^2/\omega^2)(N/N_0). \tag{2.12}$$

In the paraxial region (defined by $r \ll r_0 f$), we may make a MacLaurin series expansion (PIPES, 1958) of ε in powers of $r^2/r_0^2 f^2$. Terminating the series at the first power of $r^2/r_0^2 f^2$, we obtain (SODHA and TRIPATHI, 1977; SODHA *et al.*, 1978)

$$\varepsilon = \varepsilon_a - \varepsilon_2 r^2/r_0^2 f^2, \tag{2.13}$$

where

$$\varepsilon_a = 1 - (\omega_p^2/\omega^2)(N_{(r=0)}/N_0), \tag{2.14}$$

$$\varepsilon_2 = (\omega_p^2/\omega^2)[\partial(N/N_0)/\partial(r^2/r_0^2 f^2)]_{(r=0)}. \tag{2.15}$$

Substituting for N/N_0 from (2.3) or (2.5) or (2.7) into (2.15), we obtain

$$\varepsilon_2 = (\omega_p^2/\omega^2)\beta_p(\mathbf{E}^* \cdot \mathbf{E})_{(r=0)} \exp(-\beta_p(\mathbf{E}^* \cdot \mathbf{E})_{(r=0)}) \tag{2.16}$$

when the redistribution is caused by ponderomotive force;

$$\varepsilon_2 = (\omega_p^2/\omega^2)\beta_1(\mathbf{E}^* \cdot \mathbf{E})_{(r=0)}(1 + \beta_1(\mathbf{E}^* \cdot \mathbf{E})_{(r=0)})^{-2} \tag{2.17}$$

when the redistribution is caused by collision-dominated nonuniform heating of electrons; and

$$\begin{aligned} \varepsilon^2 = {} & 4(\omega_p^2/\omega^2) N_a^2 N_0^{-2} \varepsilon_{a(z=0)}^{1/2} \varepsilon_a^{-1/2} \beta_n E_0^2 \\ & \times (1 + 2sgN_a^2 N_0^{-2} \varepsilon_{a(z=0)}^{1/2} \varepsilon_a^{-1/2} \beta_n E_0^2)^{1/s-1} \\ & \times \{1 + [1 + 2sgN_a^2 N_0^{-2} \varepsilon_{a(z=0)}^{1/2} \varepsilon_a^{-1/2} \beta_n E_0^2]^{1/s}\}^{-2} \end{aligned} \tag{2.18}$$

when the redistribution, caused by nonuniform heating of electrons is limited by thermal conduction.

3. BEAM PROPAGATION

When $\omega^2 c^{-2} \varepsilon \gg |\nabla^2 \varepsilon|$, a plane polarized electric field in a cylinderically symmetric plasma is governed by the scalar wave equation (AKHMANOV *et al.*, 1972; SODHA *et al.*, 1976)

$$\frac{\partial^2 E}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right) + \frac{\omega^2 \varepsilon E}{c^2} = 0. \tag{3.1}$$

For a slowly diverging/converging beam, in the WKB approximation (GINZBURG, 1970), we can write (AKHMANOV *et al.*, 1972; SODHA and TRIPATHI, 1977)

$$E = A_0 \varepsilon_{a(z=0)}^{1/4} \varepsilon_a^{-1/4} \exp\left(i\omega t - i\int k\,dz - iS\right), \tag{3.2}$$

where

$$k = \varepsilon_a^{1/2}\omega/c, \tag{3.3}$$

and A_0 and S are the real amplitude and eikonal respectively. Substituting for ε and E from (2.13) and (3.2) into (3.1) and then separating the real and imaginary parts, we obtain

$$\frac{\partial A_0}{\partial z}+\frac{A_0}{2kr}\frac{\partial}{\partial r}\left(r\frac{\partial S}{\partial r}\right)+\frac{1}{k}\frac{\partial S}{\partial r}\frac{\partial A_0}{\partial r}=0, \tag{3.4}$$

$$\frac{\partial S}{\partial z}-\frac{1}{2A_0kr}\frac{\partial}{\partial r}\left(r\frac{\partial A_0}{\partial r}\right)+\frac{1}{2k}\left(\frac{\partial S}{\partial r}\right)^2+\frac{\omega^2\varepsilon_2}{2kc^2}=0. \tag{3.5}$$

The initial $(z=0)$ electric field of the laser beam is given by

$$\mathbf{E}(z=0)=EE_0\exp(-r^2/2r_0^2)\exp(i\omega t). \tag{3.6}$$

In the paraxial region $(r \ll r_0 f)$, we may express (AKHMANOV *et al.*, 1972; SODHA *et al.*, 1976) the solutions of (3.4) and (3.5) as

$$A_0=E_0 f^{-1}\exp(-r^2/2r_0^2 f^2), \tag{3.7}$$

$$S=S_a+(\omega/2c)r_0^2\varepsilon_a^{1/2}\,\mathrm{d}(\ln f)/\mathrm{d}z. \tag{3.8}$$

Substituting for A_0 and S in (3.5) and then equating the coefficients of $(r^2\varepsilon_a^{1/2}/2r_0^2 f)$ on both sides, we obtain (SODHA and TRIPATHI, 1977; SODHA *et al.*, 1978)

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\xi^2}+\frac{\mathrm{d}(\ln\varepsilon_a^{1/2})}{\mathrm{d}\xi}\frac{\mathrm{d}f}{\mathrm{d}\xi}=\frac{1}{\varepsilon_a^{1/2}}\left[\frac{1}{f^3}-\frac{r_0^2\omega^2\varepsilon_2}{c^2 f}\right], \tag{3.9}$$

where

$$\xi=zc/\omega r_0^2 \tag{3.10}$$

is the dimensionless distance of propagation. Equation (3.9) can be solved numerically by the Runge–Kutta method (SCARBOROUGH, 1966). The boundary conditions are (AKHMANOV *et al.*, 1972; SODHA *et al.*, 1976)

$$f(\xi=0)=1, \tag{3.11}$$

$$(\mathrm{d}f/\mathrm{d}\xi)_{(\xi=0)}=0, \tag{3.12}$$

which correspond to a plane wavefront at $\xi=0$.

An analysis of (3.9) in the case of a homogeneous plasma shows (SODHA *et al.*, 1978) that a laser beam gets self trapped, i.e. $f=1$, provided the initial axial intensity E_0^2 equals the self trapping intensity E_{st}^2, which is determined from the equation

$$\varepsilon_2((\mathbf{E}^*\cdot\mathbf{E})_{(r=0)}=E_{st}^2)=c^2/r_0^2\omega^2; \tag{3.13}$$

for $r_0^2\omega_p^2>(r_0^2\omega_p^2)_{crit}$, (3.13) has two roots: E_{st} (lower) and E_{st} (upper). The beam diverges monotonically, i.e. $\mathrm{d}f/\mathrm{d}\xi>0$, if $E_0^2<E_L^2$, where

$$E_L^2=c^2/r_0^2\omega^2\beta \tag{3.14}$$

is the 'linear' self trapping intensity predicted by the low field expansion of ε_2:

$$\varepsilon_e \doteq (\omega_p^2/\omega^2)\beta(\mathbf{E}^*\cdot\mathbf{E})_{(r=0)}, \tag{3.15}$$

where β is β_D, β_1 or β_n depending upon the mechanism of carrier-redistribution.

For $E_0^2 > E_L^2$, the beam oscillates periodically, i.e. the beamwidth parameter f oscillates periodically between its initial value of unity and another extremum value f_{ex}. Neglecting the axial inhomogeneity introduced by the laser beam, it can be shown that f_{ex} satisfies the transcendental equation

$$(r_0^2\omega_p^2/c^2)[\exp(-\beta_p E_0^2/f_{ex}^2) - \exp(-\beta_p E_0^2)] = 1 - f_{ex}^{-2} \tag{3.16}$$

in the case of ponderomotive force, and

$$(r_0^2\omega_p^2/c^2)[\ln(1+\beta_l E_0^2) - \ln(1+\beta_l E_0^2/f_{ex}^2)] = 1 - f_{ex}^{-2} \tag{3.17}$$

in the case of collision-dominated nonuniform heating of electrons.

4. RESULTS AND DISCUSSION

Figure 1 illustrates the dependence of βE_{st}^2 on $r_0^2\omega_p^2/c^2$ as expressed by (3.13) in the cases: (A) corresponding to the low field expansion expressed by (3.15); (B1) corresponding to collisionless plasmas in which the carrier-redistribution is caused by ponderomotive force; (B2) corresponding to collisional plasmas in which the carrier redistribution is caused by collision-dominated nonuniform heating of electrons; (B3) corresponding to collisional plasmas in which the carrier-redistribution, caused by nonuniform heating of electrons is limited by thermal conduction, and the temperature dependence of χ and ν is such that $S = 5$; and (B4) the same as (B3), but χ and ν are temperature

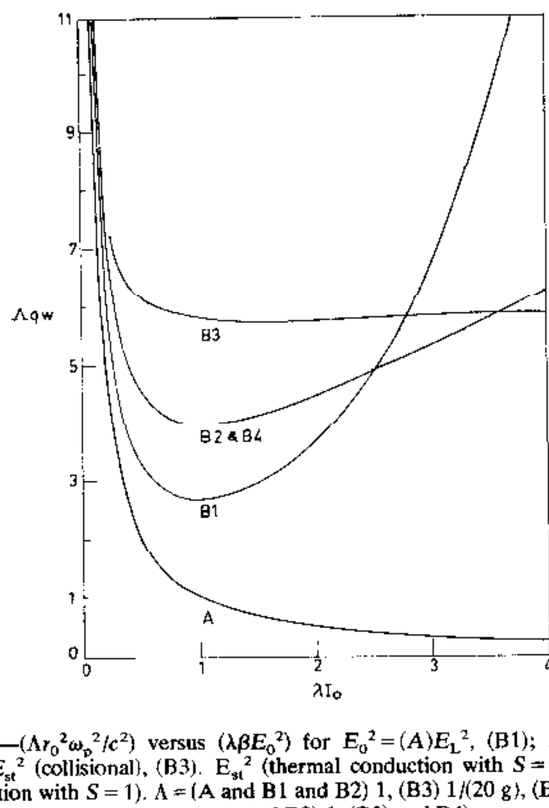

FIG. 1.—($\Lambda r_0^2\omega_p^2/c^2$) versus ($\lambda\beta E_0^2$) for $E_0^2 = (A) E_L^2$, (B1); E_{st}^2 (ponderomotive), (B2); E_{st}^2 (collisional), (B3). E_{st}^2 (thermal conduction with $S = 5$), (B4) E_{st}^2 (thermal conduction with $S = 1$). Λ = (A and B1 and B2) 1, (B3) 1/(20 g), (B4) 1/g; λ = (A and B1 and B2) 1, (B3) and B4) g.

independent so that $S = 1$. For values of $r_0^2\omega_p^2/c^2$ and βE_0^2 lying below or on the curve A, the beam diverges monotonically. For values of $r_0^2\omega_p^2/c^2$ and βE_0^2 lying in between the curve A and the curve B, the axial intensity of the laser beam oscillates periodically between the initial axial intensity E_0^2 and a minimum value $(\mathbf{E}^* \cdot \mathbf{E})_{(r=0)\min}$. For values of $r_0^2\omega_p^2/c^2$ and βE_0^2 lying on the curve B, the beam gets self trapped. For values of $r_0^2\omega_p^2/c^2$ and βE_0^2 lying above the curve B, the axial intensity of the laser beam oscillates periodically between E_0^2 and a maximum value $(\mathbf{E}^* \cdot \mathbf{E})_{(r=0)\max}$.

Figure 2 illustrates the variation of the beamwidth parameter f with the dimensionless distance of propagation ξ when the carrier-redistribution is caused by nonuniform heating of electrons limited by thermal conduction. This figure reveals that the consideration of the temperature dependence of χ/ν (represented by $S = 5$ instead of the usual $S = 1$) and that of the boundary conditions (represented by $g \gg 1$ instead of the usual $g = 1$) reduces focusing of the laser beam appreciably. (SODHA *et al.*, 1973, 1975, 1976 and TOMLIJANOVICH, 1975 had taken $S = 1$ and $g = 1$.)

The curves in the upper portion of Fig. 3 represent the variation of the extremum value f_{ex} of f (other than its initial value 1) with E_0^2 as obtained (a) solving (3.16) for f_{ex} and (b) from solving (3.9) for f corresponding to the case in which the redistribution is caused by ponderomotive force. The agreement between the analytical and numerical results is very good. The curve in the lower

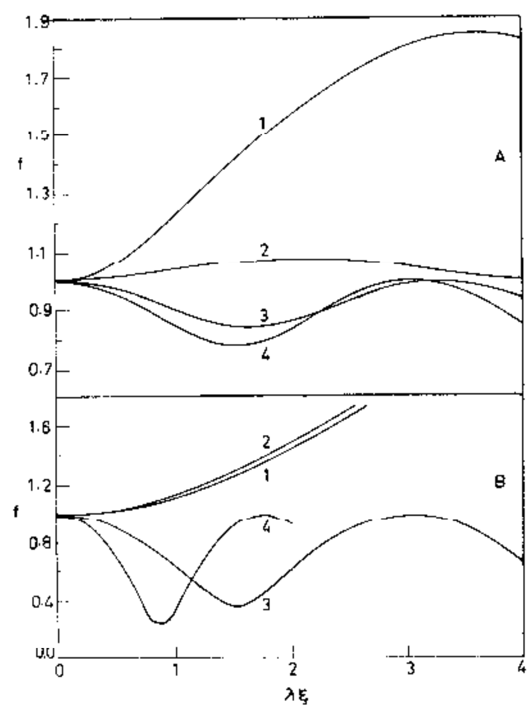

FIG. 2.—f versus $\lambda\xi$ for thermal conduction case with $s = $(1) 5, (2) 1, (3) 5, (4) 1; $g = $(A1 and A2) 10, (B1 and B2) 40, (3 and 4) 1; $\beta_n E_0^2 - $(A) 0.01, (B) 1; $r_0^2\omega^2/c^2 - $(A) 470, (B) 160; $\omega_p^2\ \omega^2 = 0.25$. $\lambda = $(A) 1, (B) 2.

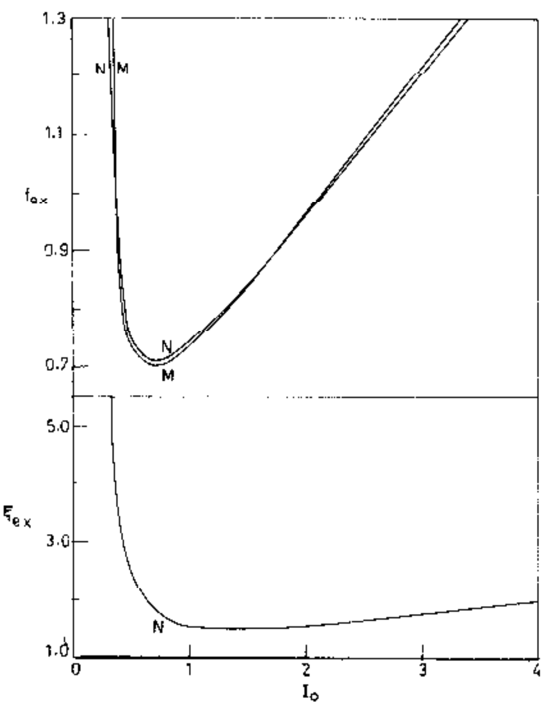

FIG. 3.—f_{ex} and ξ_{ex} versus $\beta_p E_0^2$ for $r_0^2\omega^2/c^2 = 100$; $\omega_p^2/\omega^2 = 0.04$ as obtained (a) from solving (3.16) for f_{ex}, (b) from solving (3.9) for f for ponderomotive case.

portion of Fig. 3 represents the variation of ε_{ex} (the dimensionless distance at which f becomes f_{ex}) with E_0^2 as obtained from solving (3.9) for f corresponding to the case in which the redistribution is caused by ponderomotive force.

Figures 4–7 represent the parametric variation of f with ξ corresponding to the cases in which the redistribution is caused by (A) ponderomotive force and (B) collision-dominated nonuniform heating of electrons. Figures 4, 5, 6 and 7 correspond respectively to a homogeneous plasma, an inhomogeneous plasma with exponentially decreasing electron concentration, linearly increasing concentration and periodically varying concentration. Figure 4 confirms the foregoing discussion on the qualitative behaviour of f in a homogeneous plasma. The curves in Fig. 5 show that for the electron concentration decreasing with the distance of propagation, the diffraction effect tends to predominate over the nonlinear focusing effect and consequently the beam tends to diverge monotonically even when E_0^2 exceeds E_L^2.

The curves in Figs 6 and 7 show that an intense laser beam can penetrate (ISALYEV *et al.*, 1976; SODHA and TRIPATHI, 1977) in an overdense plasma ($\omega_p > \omega$), and that the oscillations of f tend to damp out with the distance of propagation (i.e. the laser beam tends to become self trapped); the analysis being based on the WKB approximation is not valid in the resonance region where $\varepsilon_a = 0$. In an analysis of the anomalous penetration of a laser beam in an overdose plasma with linearly increasing electron-concentration, SODHA and TRIPATHI (1977) have shown that the depth of penetration (the propagation distance after

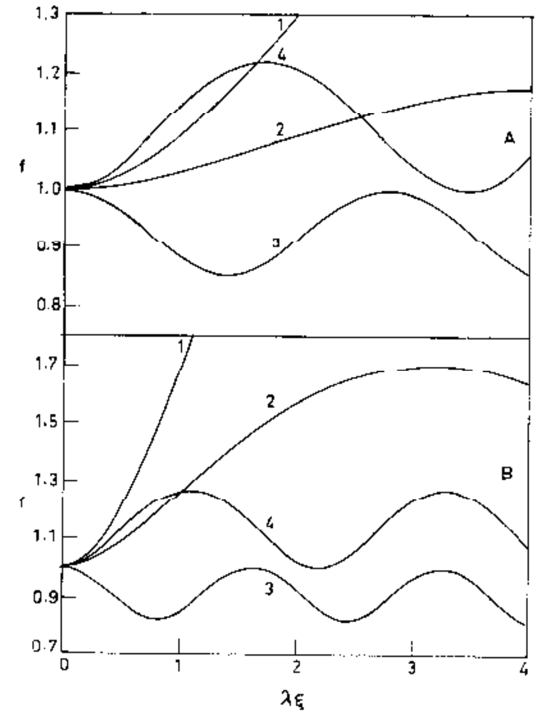

FIG. 4.—f versus $\lambda\xi$ for (A) ponderomotive case, (B) collisional case; $r_0^2\omega^2/c^2$ = (A) 16, (B) 20; $\omega_p^2/\omega^2 = 0.25$; $\beta_p E_0^2$ = (A1) 0.25, (A2) 0.3, (A3) 1.5, (A4) 3; $\beta_1 E_0^2$ = (B1) 0.15, (B2) 0.25, (B3) 1, (B4) 5. λ = (A) 1, (B) 0.5.

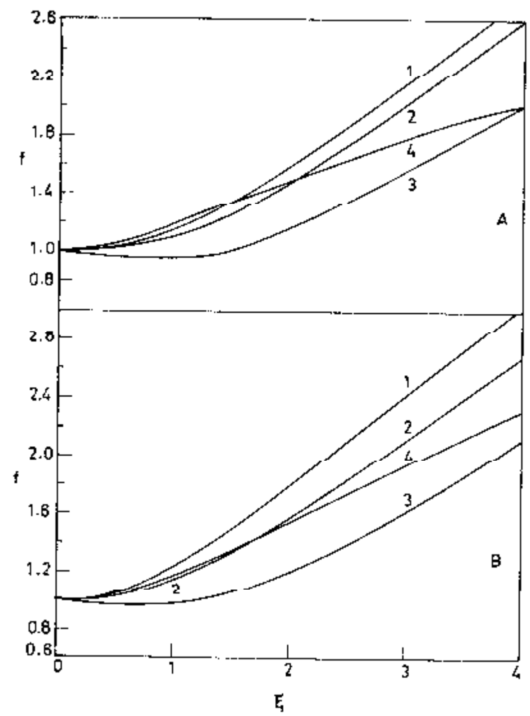

FIG. 5.—f versus ξ for (A) ponderomotive case, (B) collisional case; $r_0^2\omega^2/c^2$ = (A) 16, (B) 20; $\omega_p^2/\omega^2 = 0.25 \exp(-\xi/2)$; $\beta_p E_0^2$ = (A1) 0.25, (A2) 0.3, (A3) 1.5, (A4) 3; $\beta_1 E_0^2$ = (B1) 0.15, (B2) 0.25, (B3) 1, (B4) 5.

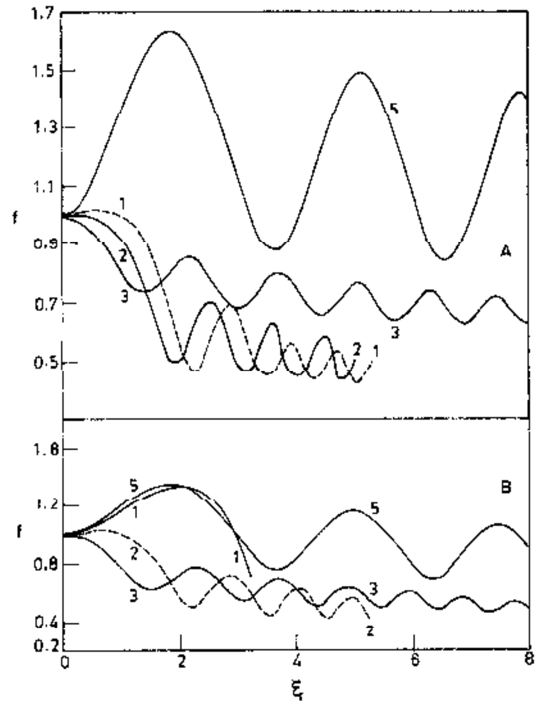

FIG. 6.—f versus ξ for (A) ponderomotive case, (B) collisional case; $r_0^2\omega^2/c^2$ = (A) 16, (B) 20; $\omega_p^2/\omega^2 = 0.25\ (1+\xi/2)$; $\beta_p E_0^2$ = (A1) 0.25, (A2) 0.3, (A3) 1.5, (A5) 6; $\beta_1 E_0^2$ = (B1) 0.15, (B2) 0.25, (B3) 1, (B5) 10.

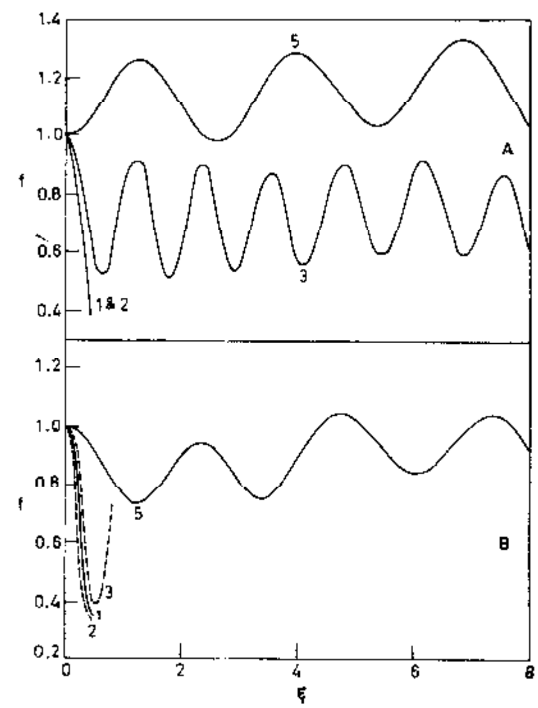

FIG. 7.— f versus ξ for (A) ponderomotive case, (B) collisional case; $r_0^2\omega^2/c^2$ = (A) 16, (B) 20; $\omega_p^2/\omega^2 = 0.25\left(4+\sin\left(\frac{\pi\xi}{4}\right)\right)$; $\beta_p E_0^2$ = (A1) 0.25, (A2) 0.3, (A3) 1.5, (A5) 6; $\beta_1 E_0^2$ = (B1) 0.15, (B2) 0.25, (B3) 1, (B5) 10.

which an electromagnetic wave becomes evanescent) increases sharply wi initial axial intensity E_0^2. We have carried out similar calculations for mechanisms as well, but the results being qualitatively alike have not presented here.

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APPENDIX

The drift velocity $\mathbf{v}$ and temperature T of electrons in a collisional plasma in the presence of beam are governed by the following momentum and energy-balance equations (SODHA *et al.*, 1975, 1976; TOMLJANOVICH, 1975).

$$m(\mathrm{d}\mathbf{v}/\mathrm{d}t) = -e\mathbf{E} - m\nu\mathbf{v} - N^{-1}\nabla_{\perp}(N\kappa_B T),$$

$$\frac{3\kappa_B}{2}\frac{\mathrm{d}T}{\mathrm{d}t} = -e\mathbf{v}^* \cdot \mathbf{E} - \frac{3m\nu\kappa_B(T-T_0)}{M} + \frac{1}{Nr}\frac{\partial}{\partial r}\left(r\chi\frac{\partial T}{\partial r}\right),$$

where N, ν and χ are respectively the electron concentration, electron collision frequency and t conductivity of the plasma in the presence of the laser beam. The three terms on the right hand (A2) correspond respectively to the energy gained by electrons from the laser beam, energy collisions, and energy lost through thermal conduction. Since the ion mass M is much larger th electron mass, $T_{ion} \simeq T_0$. Since the characteristic lengths of variations of the laser intensi electron concentration are much larger than the laser beamwidth, the axial variation of the el temperature T has been neglected in writing (A2). Since we are not considering a flowing plasn convection loss (SODHA *et al.*, 1975) is negligible in the present case. The last term on the righ side of (A1) does not contribute to the oscillatory component of $\mathbf{v}$. In the steady state, (A1) an then lead to

$$\frac{3mN\nu\kappa_B(T-T_0)}{M} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\chi\frac{\partial T}{\partial r}\right) = \frac{N\nu e^2\mathbf{E}^* \cdot \mathbf{E}}{m\omega^2}.$$

In a highly collisional plasma in which $\nu_0 \gg \chi_0 M/m\kappa_B N_0 r_0^2$, the second term in (A3) m neglected. We then obtain

$$\frac{T}{T_0}=1+\frac{Me^2\mathbf{E}^*\cdot\mathbf{E}}{3m^2\kappa_B T_0\omega^2}. \tag{A4}$$

When thermal conductivity is large such that $\nu_0 \ll \chi_0 M/m\kappa_B N_0 r_0^2$, the first term in (A3) may be neglected. We then obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\chi\frac{\partial T}{\partial r}\right)=-\frac{N\nu e^2\mathbf{E}^*\cdot\mathbf{E}}{m\omega^2}. \tag{A5}$$

In general χ and ν vary with N and T in the form (SHKAROFSKY, 1966; GINZBURG, 1970)

$$\chi/\chi_0=(T/T_0)^{n-1} \tag{A6}$$

$$\nu/\nu_0=(N/N_0)(T/T_0)^{n-s}, \tag{A7}$$

where the subscript 0 refers to absence of the laser beam; the incides n and s have been chosen to simplify the later expressions. Equation (A5) then reduces to

$$\frac{1}{nT^{n-s}r}\frac{\partial}{\partial r}\left(r\frac{\partial T^n}{\partial r}\right)=-\frac{N^2\nu_0 T_0^{s-1}e^2\mathbf{E}^*\cdot\mathbf{E}}{N_0\chi_0 m\omega^2}. \tag{A8}$$

In order to simplify the solution of (A8), we approximate the left hand side term as $\frac{1}{sr}\frac{\partial}{\partial r}\left(r\frac{\partial T^s}{\partial r}\right)$; the approximation is justified in the paraxial region, because the remainder term

$$\frac{1}{nT^{n-s}r}\frac{\partial}{\partial r}\left(r\frac{\partial T^n}{\partial r}\right)-\frac{1}{sr}\frac{\partial}{\partial r}\left(r\frac{\partial T^s}{\partial r}\right)=\frac{(s-n)}{T^s s^2}\left(\frac{\partial T^s}{\partial r}\right)^2 \tag{A9}$$

is negligible in the paraxial region where T decreases linearly with r^2. Moreover, we neglect the radial variation of N which appears in the right hand side term of (A8) and replace N by the axial electron concentration N_a. With these approximations, (A8) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T^s}{\partial r}\right)=-\frac{N_a^2\nu_0 T^{s-1}se^2\mathbf{E}^*\cdot\mathbf{E}}{N_0\chi_0 m\omega^2}. \tag{A10}$$

Substituting for $\mathbf{E}^*\cdot\mathbf{E}$ from (2.2) into (A10), and then integrating the resultant equation, we obtain

$$\frac{\delta T^s}{\partial r}=\frac{N_a^2\nu_0 T_0^{s-1}se^2E_0^2\varepsilon_{a(z=0)}^{1/2}r_0^2}{2N_0\chi_0 m\omega^2\varepsilon_a^{1/2}\quad r}[1-\exp(-r^2/r_0^2f^2)] \tag{A11}$$

As a boundary condition, it may be assumed that the plasma is not heated at and beyond the boundary located at $r=b$, so that T (at the boundary at $r=b$) $=T_0$. (A12)
Integrating (A11), we then obtain

$$T^s-T_0^s=\frac{N_a^2\nu_0 T_0^{s-1}se^2E_0^2\varepsilon_{a(z=0)}^{1/2}r_0^2}{4N_0\chi_0 m\omega^2\varepsilon_a^{1/2}}\int_{r^2/r_0^2f^2}^{b^2/r_0^2f^2}[1-\exp(-x)]x^{-1}\,dx \tag{A13}$$

In the paraxial region ($r \ll r_0 f$), we have

$$\int_0^{r^2/r_0^2f^2}[1-\exp(-x)]x^{-1}\,dx \simeq r^2/r_0^2f^2, \tag{A14}$$

and hence (A13) leads to

$$\frac{T}{T_0}=\left[1+\frac{N_a^2\nu_0 se^2E_0^2\varepsilon_{a(z=0)}^{1/2}r_0^2}{4N_0\chi_0 T_0 m\omega^2\varepsilon_a^{1/2}}(g-r^2/r_0^2f^2)\right]^{1/s}, \tag{A15}$$

where

$$g=\int_0^{b^2/r_0^2f^2}[1-\exp(-\chi)]\chi^{-1}\,d\chi. \tag{A16}$$

For a large value of b^2/r_0^2, $dg/df\sim 0$ (ABRAMOVITZ and STEGUN, 1964) and hence g may be considered a Z independent parameter evaluated at $f=1$.

Using the condition of pressure balance ($P=N\kappa_B T=$ constant), we obtain (SODHA *et al.*, 1976) the following expression for the variation of the electron concentration with electron temperature.

$$\frac{N}{N_0} \simeq \frac{2T_0}{T+T_0}. \tag{A17}$$

Using the expressions for T as derived above, we then obtain

$$\frac{N}{N_0} = \left(1 + \frac{Me^2 \mathbf{E}^* \cdot \mathbf{E}}{6m^2 \kappa_B T_0 \omega^2}\right)^{-1} \tag{A18}$$

when $\nu_0 \gg \chi_0 M / m\kappa_B N_0 r_0^2$, and

$$\frac{N}{N_0} = 2\left\{1 + \left[1 + \frac{N_a^2 \nu_0 s e^2 E_0^2 \varepsilon_{a(z=0)}^{1/2} r_0^2}{4N_0 \chi_0 T_0 m\omega^2 \varepsilon_a^{1/2}} \left(g - r^2/r_0^2 f^2\right)\right)\right]^{1/s}\right\}^{-1} \tag{A19}$$

when $\nu_0 \ll \chi_0 M / m\kappa_B N_0 r_0^2$.