Optik 50 (1978) No. 3, 243–247

Self induced transparency of a two-frequency pulse

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Received 17 October 1977

Abstract

The two modes of a two frequency electromagnetic pulse are parametrically coupled in a medium which is resonantly absorbing at these two frequencies. It is found that the pulse can pass undistorted provided the initial electric-field-amplitude E_{l0} be large enough, and the effective refractive index $(\eta_l + 4\pi\hbar\omega_l n_l/\eta_l E^2_{l0})$ be the same for both the modes.

Inhalt

Selbstinduzierte Transparenz bei einem Puls mit zwei Frequenzen. Zwei Moden werden bei Resonanzabsorption parametrisch in einem Medium gekoppelt. Wie sich zeigt, kann deren Impuls unverzerrt hindurchtreten, vorausgesetzt die Amplitude E_{l0} der einfallenden Welle ist ausreichend hoch und die effektive Brechzahl $(\eta_l + 4\pi\hbar\omega_l n_l/\eta_l E^2_{l0})$ ist für beide Moden die gleiche.

1. Introduction

The phenomenon of self-induced-transparency (SIT) has been widely investigated [1, 2] and shown to be related to a number of optical resonance phenomena in two-level systems [3]. Recent investigations on two photon SIT [4, 5], relativistically induced transparency in plasmas [6], dispersive parametric interactions [7] and SIT type solutions for plasma excitations [8] clearly indicate the plausibility of SIT in poly-level systems. It is now possible and important to study SIT of a poly-chromatic pulse in a poly-level system.

The present investigation deals with SIT of a two-frequency electromagnetic pulse through a medium which is resonantly absorbing at these two frequencies. (By a two-frequency pulse is meant a pulse whose spectrum has two distinct peaks at some two frequencies.) Each frequency-portion of this two-frequency pulse will, for convenience, be called a mode. By assuming the coupling between the two modes to be weak and parametric [7, 9] in nature, modified coupled [SIT] equations are obtained for each mode. Successive Born approximation technique is employed to derive soliton-type solutions of these equations. It is found that the pulse can pass undistorted provided the initial electric-field-amplitude E_{l0} be large enough, and the effective refractive index $(\eta_l + 4\pi\hbar\omega_l n_l/\eta_l E_{l0}^2)$ be the same for both the modes.

2. Basic equations

Let there be a medium containing two types (say $l = A$ and $l = B$) of two-level systems. When uncorrelated, both these types individually have pseudo-spin representations [3] in terms of the Bloch vectors $\boldsymbol{B}_l = \langle u_l, v_l, w_l\rangle$. Let this medium be irradiated by a two-frequency electromagnetic pulse with electric field

$$\boldsymbol{E} = \boldsymbol{E}_A + \boldsymbol{E}_B\,,$$

$$\boldsymbol{E}_l = \mathrm{Re}\,\langle 1, -i, 0\rangle\, \varepsilon_l \exp\left[i\,(\omega_l' t - k_l z + \Phi_l)\right], \tag{1}$$

where

$$\omega_l' = (\text{upper state energy} - \text{lower state energy})_l/\hbar + \delta\omega_l\,,$$

and the mode-envelopes ε_l vary with the propagation distance z as well as time t. The electric fields $\boldsymbol{E}_l$ drive $\boldsymbol{B}_l$, in the rotating coordinate systems, according to the Bloch equations [3]

$$\frac{d\,\boldsymbol{B}_l}{dt} = \Omega_l \times \boldsymbol{B}_l, \tag{2}$$

where

$$\Omega_l = \langle -2p_l\varepsilon_l/\hbar,\, 0,\quad \delta\omega_l + \partial\Phi_l/\partial t\rangle,$$

$$p_l = \left|\langle \text{upper } l \text{ state} \,\middle|\, e\boldsymbol{r} \,\middle|\, \text{lower } l \text{ state}\rangle\right|,$$

and the longitudinal and transverse relaxation times T_{l1} and T_{l2} have been assumed to be much greater than the pulse-duration. If n_l is the number of l type particles, the corresponding pseudo-polarization vector [2, 3] is

$$\boldsymbol{P}_l = \mathrm{Re}\,\langle 1, -i, 0\rangle\, n_l p_l\,(u_l + iv_l) \exp\left[i\,(\omega_l' t - k_l z + \Phi_l)\right]. \tag{3}$$

Let $|\omega_A - \omega_B| \gg 0$. Let $\eta_l = (ck_l/\omega_l)$ denote the linear background refractive index. The two modes will be assumed to be sharply resonant, unmodulated, transversely homogeneous and with slowly varying envelopes. The two modes may be coupled due to some internal or external source field at frequency $(\omega_A' \pm \omega_B')$ in the medium. This coupling will be assumed to be weak and parametric in nature and will be introduced through phenomenological real coupling coefficients γ_l. Then it can be shown that ε_l are governed by the modified coupled SIT equations [1, 2, 9] as follows:

$$\left(\frac{\partial}{\partial z}+\frac{\eta_l}{c}\frac{\partial}{\partial t}\right)\varepsilon_l=\left(\frac{2\pi\omega_l n_l p_l}{c\eta_l}\right)v_l+\gamma_l\varepsilon_\lambda, \tag{4}$$

$$\frac{\partial v_l}{\partial t}=\left(\frac{2p_l}{\hbar}\right)\varepsilon_l w_l\,, \tag{5}$$

$$\frac{\partial w_l}{\partial t}=-\left(\frac{2p_l}{\hbar}\right)\varepsilon_l v_l\,, \tag{6}$$

where $\lambda = B$ for $l = A$ and $\lambda = A$ for $l = B$. Note that, but for the terms involving v_l, eq. (4) describe the problem of parametric interaction [9] between the two modes. The coupling coefficients γ_l as well as the source-field-amplitude on which they depend linearly are here assumed to be known constant parameters.

In terms of the area defined as

$$\Delta_l=\left(\frac{2p_l}{\hbar}\right)\int\limits_{-\infty}^{t}\varepsilon_l dt\,, \tag{7}$$

we have

$$\varepsilon_l=\left(\frac{\hbar}{2p_l}\right)\frac{\partial}{\partial t}\Delta_l\,, \tag{8}$$

$$v_l=\sin\Delta_l\,, \tag{9}$$

$$w_l=\cos\Delta_l. \tag{10}$$

The six coupled equations (4–6) then reduce to the following two coupled equations for Δ_l.

$$\left(\frac{\partial^2}{\partial z\partial t}+\frac{\eta_l}{c}\frac{\partial^2}{\partial t^2}\right)\Delta_l=\left(\frac{4\pi\omega_l n_l p_l^2}{\hbar c\eta_l}\right)\sin\Delta_l+\gamma_l\frac{\partial}{\partial t}\Delta_\lambda. \tag{11}$$

3. Soliton type solutions

To simplify the solutions of eqs. (11), only distortionless or the socalled soliton-type solutions [10, 11] will be sought for. Eqs. (11) are reduced to the modified coupled sine-Gordon equations

$$\frac{\partial^2}{\partial\tau_l^2}\Delta_l=-\mu_l^2\sin\Delta_l+\beta_l\frac{\partial}{\partial\tau_l}\Delta_\lambda, \tag{12}$$

where

$$\mu_l^2 = \left(\frac{4\pi\omega_l n_l p_l^2}{\hbar\eta_l}\right)\left(\frac{c}{\eta_l V_l} - 1\right)^{-1},$$

$$\tau_l = (t - z/V_l),$$

$$\beta_l = (c\gamma_l/\eta_l). \tag{13}$$

Eqs. (12) may be conveniently solved by the successive Born approximation technique as follows. Consider the coupling between the two modes to be weak enough to be considered as a small perturbation. Denote the m^{th} Born approximation results by the subscript m to Δ_l, ε_l, μ_l, τ_l, V_l and subscript $(m-1)$ to $(\partial\Delta_\lambda/\partial\tau_l)$. Then eqs. (12) reduce to

$$\frac{\partial^2}{\partial\tau_{lm}^2}\Delta_{lm} = -\mu_{lm}^2 \sin\Delta_{lm} + \beta_l \frac{\partial\Delta_{\lambda(m-1)}}{\partial\tau_{l(m-1)}}. \tag{14}$$

These are to be solved along with the boundary conditions

$$\lim_{|z|\to\infty} \Delta_l = 0. \tag{15}$$

Assuming $\Delta_{\lambda(m-1)}$ to be known and neglecting τ dependences of V_{lm}, eq. (14) is solved to obtain

$$\Delta_{lm} = 2\sin^{-1}\tanh(\mu_{lm}\tau_{lm}) + \beta_l \int_{-\infty}^{\tau_{\lambda(m-1)}} \Delta_{\lambda(m-1)}\, d\tau_{\lambda(m-1)}. \tag{16}$$

Consequently, according to eq. (8), we obtain

$$\varepsilon_{lm} = \left(\frac{\hbar\mu_{lm}}{p_l}\right)\operatorname{sech}(\mu_{lm}\tau_{lm}) + \left(\frac{\hbar\beta_l}{2p_l}\right)\Delta_{\lambda(m-1)}(\tau_{\lambda(m-1)}). \tag{17}$$

If $\quad \varepsilon_{lm}(t=0, z=0) \simeq \varepsilon_l(t=0, z=0) = E_{l0},$

then $\quad \mu_{lm} = (p_l E_{l0}/\hbar) - (\beta_l/2)\,\Delta_{\lambda(m-1)}(0). \quad (18)$

and consequently,

$$V_{lm} = c\left\{\eta_l + \frac{4\pi\hbar\omega_l n_l}{\eta_l E_{l0}^2}\left(1 - \frac{\hbar\beta_l}{2p_l}\frac{\Delta_{\lambda(m-1)}(0)}{E_{l0}}\right)^{-2}\right\}^{-1}. \tag{19}$$

4. Conclusion

Note that V_{lm} are independent of τ, as required, provided $E_{l0} \gg (\hbar\beta_l/2p_l)\,\Delta_{\lambda(m-1)}(0)$. With this condition satisfied, V_{lm} becomes independent of m also. For the whole of the pulse to propagate undistorted through the medium, the following three conditions should be satisfied in addition to the above-mentioned inequality condition. The first and the most important condition is that both the modes should have the same velocity V which means that the effective refractive index $(\eta_l + 4\pi\hbar\omega_l n_l/\eta_l E_{l0}^2)$ should be independent of l. The second condition is that the product of the coupling coefficient and area $\beta_l \Delta_{\lambda(m-1)}$ (any τ) should be negligibly small which means that the coupling between the two modes should be sufficiently weak. The third condition is that μ_{lm} should be the same for both the modes which means that $p_l E_{l0}$ should be independent of l. When any of these conditions fails, each mode may separately pass undistorted through the medium, but the two-frequency pulse as a whole gets distorted.

The foregoing analysis may be generalized for a multi-frequency (polychromatic) pulse through a resonantly absorbing poly-level medium.

The author is grateful to Professor M.S. Sodha, Dr. D.P. Tewari and Dr. V.K. Tripathi for encouraging discussions on the subject. This work was partially supported by N.C.E.R.T. (India) and N.S.F. (U.S.A.).

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